Tribocharging and the Finite Thickness Interface

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Abstract—**When two uncharged solid materials are brought together and then separated, a charge often occurs on the surfaces of the two materials where they had been in contact; and this charging is referred to as contact charging, triboelectric charging or simply as tribocharging. Why this charging occurs is still a question open for scientific discussion. In this paper a model for understanding the mechanism of tribocharging is presented based on 1) a solid-solid contact interface, 2) the charge flux equation, 3) the concept that all materials have some free-electron charges – many in metals, very few in insulators – and 4) the concept – for two materials in contact at an interface and in equilibrium – that the total electrical charge flux J must always be zero everywhere in both materials. It is shown that across the interface this J = 0 restriction still allows free-electron movement when under the action of a force. Collision forces, occurring in a free-electron diffusional gradient, drive a free-electron diffusional current density J_D which continually moves free-electrons across the interface from the material of the higher free-electron charge density and into the material of lower free-electron charge density. This movement of free-charges sets up an electric field E in the same region which gives rise to an E-field driven current density J_E continually driving the free-electrons back in the direction from which they came. In equilibrium $J_D = -J_E$ everywhere within the interface region, while J = 0 everywhere. When the two materials are separated free-electron charges are inevitably left in the material having the lower free-electron charge-density giving it a negative charge while the absence of some of these free-electron charges in the higher free-electron charge-density material leaves it with a positive charge. The model gives a simple and clearly understandable explanation of tribocharging. Furthermore, for two materials having only a slight difference between their free-electron charge densities the thickness of the interface can be predicted. Results from the model describe the general features found in the hydrophobic lower half of a typical triboelectric series table. However, real-world surfaces can be either hydrophilic (i.e., water loving and adsorb moisture and pollutants), hydrophobic (water hating) or oxidative; and the hydrophilic surfaces found in the upper half of a typical triboelectric series table will require a further refinement to the model based not on a solid-solid contact interface but rather on a solid-adsorbed-liquid-solid interface.**

Keywords–tribocharging; triboelectric; contact; interface; hydrophilic; hydrophobic; conductor; semiconductor; insulator; Ohm's Law; Debye length; interfacial thickness; charges; free-electrons; interfacial potential; thunderclouds; hailstones; ice crystals;

I. INTRODUCTION

When two uncharged solid materials are brought together and then separated a charge often occurs at the interface where the contact had been made [1]. This charging is referred to as contact charging; and if any rubbing of the contacting surfaces also occurs, it is referred to as triboelectric charging. Often the term triboelectric charging is reduced to the single word – tribocharging – and is used as a general descriptive term for any contact charging process even when rubbing does not occur or is not knowingly initiated. Experimentally, different materials can be contacted and separated, and the magnitude of their charged surfaces measured and then listed in a table. When two materials are separated the material receiving a positive charge is listed above the material receiving the negative charge. Eventually a series of tests can be tabulated with the material receiving the highest positive charge being listed at the top of the table and the material receiving the highest negative charge being listed at the bottom of the table. A table thus developed is referred to as a triboelectric series and many such triboelectric series have been developed over the years [1][2][3][4]. A generic representation of a triboelectric series is shown in Table 1. The Pareto principle [5], aka the 80–20 rule, often used in economics seems to have an analog to any triboelectric series. Whenever a table is presented, about 80% of researches will agree with the listing while 20% will have measured data that is in conflict. Thus, any triboelectric series is useful as a guide but can not be taken as an exact scientific listing [4].

TABLE 1: GENERALIZED GENERIC REPRESENTATION OF A TRIBOELECTRIC SERIES

Classification	Surface Property	Example Materials
Organic, typically polar hydrocarbons	Hydrophilic	Asbestos ⋮ Hair, Nylon
⋮	⋮	⋮
Inorganic, crystalline	Oxidative (corrosion)	Metals, many Ceramics
⋮	⋮	⋮
Organic, typically non-polar hydrocarbons and fluorocarbons	Hydrophobic	Polyethylene ⋮ Teflon, Silicone

TABLE 2: TRIBOELECTRIC SERIES OBSERVATIONS

#	Description
1	Metal to metal tribocharging is usually small (low amounts of charge on each metal)
2	Glass and nylon are usually high in the series (tend to receive a high positive charged)
3	Silicone and Teflon are usually low in the series (tend to receive a high negative charged)
4	Insulator to insulator charging tends to be the least repeatable.
5	Insulators when tribocharged have regions of charge that are both positive and negative
6	Hydrophilic materials (which adsorb moisture) tend to be found at the top of the series
7	Hydrophobic materials (which do not adsorb moisture) tend to be found at the bottom of the series

Some further description of Table 1 is warranted. Referring to Table 1 in a triboelectric event the further apart the two contacting materials appear in the table the higher will be the charge after separation with the material listed higher in the table receiving the positive charge and the material listed lower in the table receiving the negative charge. Examples: A nylon-metal contact results in nylon(+) and metal(-) whereas a metal-silicone contact results in metal(+) and silicone(-). If properly cleaned a metal-metal, nylon-nylon or silicone-silicone contact results in very little charging whereas a nylon-silicone contact results in very high charge transfer with nylon(+) and silicone(-). In air many surfaces adsorb some moisture and will be covered by a layer of water from less than a monolayer on a hydrophobic surface to a macroscopic thin film on a hydrophilic surface (see [1], pp. 27-28).

In general, tribocharging is poorly understood [1][6][7]. However, there have been some general observations [1][4][6][8] regarding any triboelectric series; and these are listed in Table 2. Likewise there have also been some general observations [1][4][6][8][9] on the tribocharges produced by tribocharging, and these are summarized in Table 3.

Ohm's Law applies to a linear, homogeneous and isotropic material, but there is little hope in using Ohm's Law to build a

TABLE 3: GENERAL OBSERVATIONS ON TRIBOCHARGES

#	Description
1	Tribocharges have been found to some depth in some insulators
2	Electrons are the main specie active in some tribocharging events
3	Chemical reaction at the surface can be responsible for charging on some surfaces
4	Material removal and deposition at the surface can cause tribocharging
5	Tribocharging can be temperature and humidity dependent

theory of tribocharging. The reason is, in the past, the tribocharges have been assumed to occur at a zero thickness interface and Ohm's Law only predicts a voltage drop across a region of finite thickness. On the other hand if tribocharges do occur to some depth then the material is no longer homogeneous and isotropic so Ohm's Law no longer applies within the interface depth. The present paper addresses this Ohm's Law conflict and offers a solution to the problem.

Any theory of tribocharging will have to explain both the triboelectric series observations listed in Table 2 and the general observations on tribocharges listed in Table 3. This paper looks at a new, but very realistic, approach to developing an understanding of tribocharging based on the charge flux equation. As will be discussed, the theory is found to apply to the hydrophobic lower portion of Table 1.

II.   SOLID-SOLID MATERIALS MODEL

A solid can be classified as either a conductor, semiconductor or an insulator depending on its electrical volume resistivity (see [4], p. 53) or its electrical conductivity; and one such classification is shown in Table 4. Silver has the highest electrical conductivity of all the metals; $\sigma \approx 6 \times 10^7$ S/m. In practice most solid materials fall into either the conductor or insulator category with the exception in the semiconductor industry where manufactured semiconductors have an electrical conductivity determined by the impurity dopant concentration [10]. In order to understand what happens at an interface between two initially uncharged solid materials, a model must first be defined for the materials. The model chosen here assumes each solid material –when in equilibrium with a perfectly non-conductive gas incapable of electron attachment– is homogeneous, linear and isotropic. It is further assumed that initially the solid materials are charge neutral being made up of atoms and molecules that are charge neutral.

The assumption that electrons take part in tribocharging is not without experimental evidence [11]. In this paper it is simply assumed there is some fraction f_{ie} of the atoms in any solid which can be treated as if each atom, within this fraction, is a positive-charged ion and a negative-charged free-electron. It is also assumed that this fraction f_{ie} is near unity in metals (Drude model [12]), is approximately equal to the impurity dopant atom concentration to total atom concentration ratio in a semiconductor [10], and is of a very small value in a good insulator {[13] see p. 212-213}. It is known that for insulators a small continuous current is found to flow after the application of a steady-state electric field and at low fields this current is found to be ohmic {[13] see p. 26 and p. 207}.

In the model chosen here it is assumed that for an isolated material the positive-ion-charge density is ρ_{0+} and the negative free-electron charge density is ρ_{0e} and for charge neutrality $\rho_{0+} = -\rho_{0e}$ everywhere when the solid is isolated. It is further assumed the atoms cannot move from their equilibrium positions so that $\rho_+ = \rho_{0+}$ is constant throughout the solid material even when the material is not isolated. However, even though the negative free-electron charge density is $\rho_e = \rho_{0e}$ everywhere for an isolated material, it is assumed ρ_e –being composed of free electrons– can freely vary to satisfy any imposed conditions at a solid-solid interface. These assumptions are explicitly listed in Table 5.

TABLE 4: ELECTRICAL CLASSIFICATION OF MATERIALS (ANALOGOUS TO [4])

Classification	Range
Conductors	$10^2 \leq \sigma \leq 10^8$ S/m
Semiconductors	$10^{-4} \leq \sigma \leq 10^2$ S/m
Insulators	$\sigma \leq 10^{-4}$ S/m

TABLE 5: ASSUMPTIONS IN THE (SOLID-SOLID INTERFACE) MODEL

#	Assumption
1	All solid materials are made up of atoms and molecules
2	An isolated solid surrounded by a perfectly non-conducting gas is homogeneous & isotropic
3	Some fraction f_{ie} of the atoms behave as if each atom were a fixed +ion and a free-electron
4	In #2 and #3 the intrinsic charge densities in the isolated solids are ρ_{0+} for ions and ρ_{0e} for free-electrons
5	The (isolated) charge densities in #4 are $\rho_+ = \rho_{0+}$ and $\rho_e = \rho_{0e}$ and for neutrality $\rho_{0+} = -\rho_{0e}$
6	For 2 solids in contact the charge densities are $\rho_+ = \rho_{0+}$ everywhere, but ρ_e is defined by the interface conditions
7	Total charge flux $\mathbf{J} = 0$ everywhere, for isolated materials and also when in equilibrium contact
8	Total charge flux in the solid is $\mathbf{J} = \mathbf{J}_+ + \mathbf{J}_e$ but in solids +ions can not move: $\mathbf{J}_+ = 0$, so $\mathbf{J} = \mathbf{J}_e$
9	For two solid materials in equilibrium contact, ρ_e must be continuous across the interface
10	The charge flux equation [15] gives the total charge flux $\mathbf{J}$
11	At the contact interface a region develops in both solids that is no longer homogeneous & isotropic
12	The developed region in #11 has a thickness δ_{IF} and constitutes a finite thickness interface
13	The dielectric constant (ϵ_r) – of the two contacting materials – does not change in the interface region δ_{IF}
14	The solid-solid contact is between hydrophobic surfaces (no adsorbed moisture)
15	There is no generation or recombination of charge occurring anywhere including the interface region δ_{IF}

III. CHARGE FLUX EQUATION

In tribocharging two uncharged solids are brought together in intimate contact as depicted in Fig. 1. Later, when the solids are separated some charges are found on the contacted surfaces. As a result, some charge must have moved across the contacting area in a unit of time – that time being equivalent to the time of contact. Hence, to understand tribocharging requires an understanding of how charges are transported across the contacting surfaces. Any transport of charge crossing a unit area at a right angle to that area in a unit time is known as a charge flux or current density [14].

A. Free-electron Charge Flux

Charge flux is represented by the vector symbol $\mathbf{J}$ and the charge flux equation [15] gives the charge transport of multiple species across a unit area per unit time, i.e., $\mathbf{J} = \sum \mathbf{J}_i$. In the present paper only two charged species –free-electrons and bound +ions– are assumed to be the species of interest. The charge flux equation can be developed based purely on the calculus of a gradient across a volume element [16]. However, the charge flux equation developed from the classical physics model of collisions [15] will be used here and when only electrons are free to move the charge flux equation reduces to the equation for free-electron charge flux $\mathbf{J} = \sum \mathbf{J}_i \rightarrow \mathbf{J_e}$ and is given by

$$\mathbf{J_e} = \sigma \mathbf{E} - D \nabla \rho_e - G \rho_e \nabla T \quad \text{(Electron Charge Flux)} \quad (1)$$

where σ is the electrical conductivity of the material, and – based on assumption 8 in Table 5, free-electrons are the only specie capable of moving– σ is due entirely to electron conduction. In (1) $\mathbf{E}$ is the electric field at any point where $\mathbf{J}$ is evaluated. The other terms in (1) are the charge density

$$\rho_e = q_e n_e = s_e q_0 n_e = -q_0 n_e \quad \text{(Electron Charge Density)} \quad (2)$$

where $q_e = s q_0 = s_e q_0$ is the charge of the specie (free-electrons here) where $s = \pm 1$ depending on the sign of the charge, (here $s = s_e = -1$), and q_0 is a positive value equal to the absolute value of the electron charge or equal to the charge on a positron, $q_0 \approx +1.6 \times 10^{-19}$ coulombs, and n_e is the number density of the free-electrons at the point where $\mathbf{J_e}$ is evaluated. Note: the present author has chosen to keep s in the equations because in more complicated situations where many species are involved (e.g., liquids and gases) it is easier to keep track of each specie with its sign observable. Conductivity in (1) is

$$\sigma = s_e^2 q_0 n_e b_e = s_e \rho_e b_e \quad \text{(Electrical Conductivity)} \quad (3)$$

where b_e (always a positive number) is the mobility of the electrons in the solid given by

$$b_e = q_0 \tau_e / m_e \quad \text{(Electron Mobility)} \quad (4)$$

where τ_e is the average electron collision time and m_e is the mass of the electron, $m_e \approx 9.11 \times 10^{-31}$ kg. Note: the E-field drift velocity of any specie is defined as {in [15] see Eq (9) and (14a)} $\mathbf{v_{dE}} = s q_0 \frac{\tau}{m} \mathbf{E} = s b \mathbf{E}$, so for free-electrons

$$\mathbf{v_{dE}} = s_e q_0 \frac{\tau_e}{m_e} \mathbf{E} = s_e b_e \mathbf{E} = -b_e \mathbf{E} \quad \text{and as expected the}$$
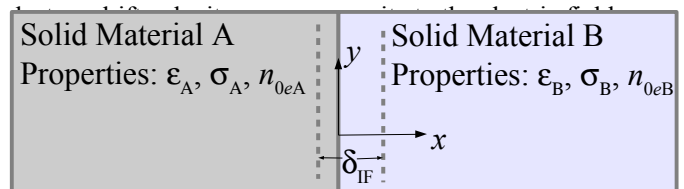
Fig. 1: Two solids in contact at the x = 0 plane.

where k is Boltzmann's constant, $k \approx 1.38 \times 10^{-23}$ J/K, and T is the temperature at the evaluation point. Equation (5) is known as both the Einstein relation [17], [18] and as the Nernst-Einstein equation [19]. Finally the electron thermophoresis coefficient in (1) is

$$G = k b_e / q_0 = D / T \quad \text{(Thermophoresis Coefficient)} \quad (6)$$

and (6) shows the thermophoresis coefficient G depends on the electron mobility b_e.

A second set of equations similar to (1) through (6) can be written for the positive-charged ions by replacing the subscript e with a + sign. However, (3) (5) and (6) contain the mobility (4) and using assumption 8 in Table 5 the +ion cannot move in the solid and as a result its mobility is zero so (3) (5) and (6) will all have a value of zero for the +ion and (1) reduces to

$$\mathbf{J}_+ = 0 \quad \text{(Only valid in a solid)} \quad (7)$$

as the charge flux equation for the +ion. But everywhere $\mathbf{J} = \mathbf{J}_+ + \mathbf{J}_e$ and since, as just discussed for solids, $\mathbf{J}_+ = 0$ the total charge flux is simply $\mathbf{J} = \mathbf{J}_e$ which mathematically expresses assumption 8 in Table 5. At first glance it may seem a bit annoying to see (7) obtain the status of a numbered equation, but it will be reiterated in the Discussion Section that (7) is crucial in understanding tribocharging.

B. Understanding the Charge Flux Equation

The charge flux equation (1) may look daunting at first view, but it is very easy to understand.

1) Ohm's Law:

Consider a solid material –specifically a solid material to be considered later for tribocharging– which is surrounded by a non-conducting and non-interacting gas at a temperature T. If the solid is homogeneous and isotropic, then $\nabla \rho_e = 0$ and, if it has come to thermal equilibrium with the surrounding gas, it is at a constant temperature T so $\nabla T = 0$ and (1) reduces to

$$\mathbf{J} = \sigma \mathbf{E} \quad \text{(Ohm's Law; } \nabla \rho_e = 0 \text{ and } \nabla T = 0) \quad (8)$$

which is the electric field form of the expression for Ohm's Law.

2) Field & Circuit Forms of Ohm's Law:

Some readers may be unfamiliar with the field form of Ohm's Law as given in (8). The field form can be related to the circuit form with the following discussion. Consider a resistor of length L placed symmetrically along the x axis and positioned from $x = 0$ to $x = L$ as shown in Fig. 2. Next, allow a current I to pass through the resistor as shown in Fig. 2. But $I = dq/dt = d(s q_0)/dt = s(dq_0/dt) = s I_0$ where the current I_0 is a positive quantity. But the charge flux $\mathbf{J}$ is the movement of charge per unit time per unit area which for the resistor of cross-sectional area A is simply the current I per unit area A.
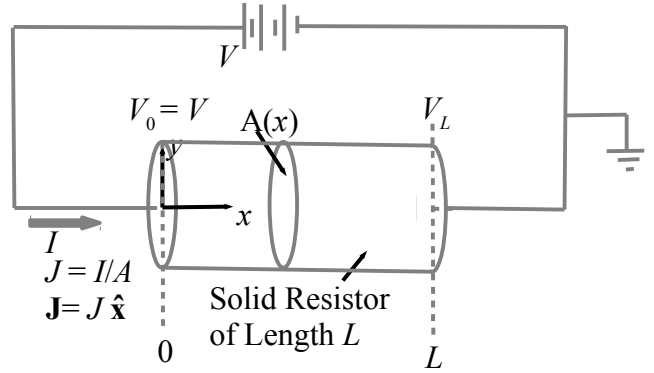

Fig. 2: Resistor of length L used to show how the field form of Ohm's Law converts to the circuit form.

Hence if $A(x)$ is the cross-section area of the resistor at any point x then $J = s I_0 / A(x)$. For the axis of the resistor being placed along the x-direction $\mathbf{J}$ and $\mathbf{E}$ have only an x-component, so $\mathbf{J} = J \hat{\mathbf{x}}$ and $\mathbf{E} = -\nabla V$ reduces to $\mathbf{E} = -dV/dx \hat{\mathbf{x}}$ and (8) can be written simply as $J = -\sigma dV/dx$. Rearranging this last equation gives $-dV = J dx/\sigma = s I_0 dx/[\sigma A(x)] = s I_0 dR$ where $dR = dx/[\sigma A(x)]$. Set the potential to V at the $x = 0$ end of the resistor and V_L at the distance $x = L$ as shown in Fig. 2. Then, integrating dV from V to V_L while integrating dR from 0 to R (which requires integrating $dx/[\sigma A(x)]$ from 0 to L) gives $R = \int_0^L dx/[\sigma A(x)]$ and the integration gives $-(V_L - V) = s I_0 R = IR$. Note that this last equation can be written as $V - V_L = IR$. and if V_L is set to reference ground ($V_L = 0$) as shown in Fig. 2, then $V = IR$ which is the circuit form of Ohm's Law. Although in general $R = \int_0^L dx/[\sigma A(x)]$ when A has a constant cross-section {as depicted in Fig. 2 – i.e., $A \neq A(x)$} then R becomes the more familiar form $R = L/(\sigma A)$.

3) Isolated solids:

Consider a solid material –like a material to be considered later for tribocharging– which is surrounded by a non-conducting gas at a temperature T. If the solid (call it material A) is homogeneous and isotropic, then $\nabla \rho_e = 0$ and if it has come to equilibrium with the surrounding gas it is at a constant temperature T so $\nabla T = 0$ and (1) reduces to (8) so the material satisfies the Ohm's Law criteria.

Next, take a second solid material which is also homogeneous and isotropic and repeat the same arguments. Since the arguments are the same, this second solid (call it material B) also satisfies Ohm's Law when completely surrounded by the non-conducting gas.

4) Connected Solids (zero-thickness methodology):

Connect the two materials described above in Section 3)Isolated solids: together. Both material A and material B were originally defined as homogeneous and isotropic in the

isolated state and both followed Ohm's Law. Has anything changed?

5) Zero Thickness Interface (ZTI) Postulate:

Let the following (conventional) postulate be made:

- Assume as a postulate that when two solid materials are connected together that both materials *remain* homogeneous and isotropic (including electrically homogeneous and isotropic).

Consider now Fig. 1 and let the materials fill all space in the *yz*-plane. For the above ZTI postulate Ohm's Law also follows when they are connected together. Namely, each material has a well defined electrical conductivity throughout the material. When moving a point of interest (say along the *x* direction from the first material (A) occupying all *x* < 0 and into the second material (B) occupying all *x* > 0), the conductivity is well defined at all points except for the jump at *x* = 0. The point *x* = 0 is the interface and it does not present a problem as calculus teaches how to examine a point approaching from *x* = 0₋ or from *x* = 0₊ [20]. Everything is well understood using Ohm's Law, and it is logically clear that the interface must have zero thickness otherwise an interface of finite thickness would invade the territory of one or both materials and invalidate the zero thickness interface (ZTI) postulate.

What is quite useful about the ZTI postulate is that it makes Ohm's Law valid everywhere and much of electrical engineering is practiced today using Ohm's Law albeit the circuit form *V* = *IR* rather than the field form given in (8). What Ohm's Law does not do –and cannot do– is produce any mechanism for charge transfer when two materials are separated since everything is well specified and therefore well known at all time in either material all the way up to the zero thickness interface.

The ZTI postulate does not prove the existence of a zero thickness interface, but rather it shows that if, and only if, the solids remain homogeneous and isotropic that a zero thickness interface will exist. The question of a possible need for a non-zero thickness interface was discussed by James Clerk Maxwell in regards to the concept of surface charge density, and his key ideas are reviewed below.

6) Maxwell's Choice:

James Clerk Maxwell's contributions to science are numerous but his crowning achievement was the publication of his two volume TREATISE ON ELECTRICITY AND MAGNETISM [21][22] in which he combined electricity and magnetism to give the framework of electromagnetic theory. Lesser known in his book was his struggle to decide how to define surface charge density. He presented two possibilities (see [21], page 72). In one scenario he reasoned that charges had no mass and needed no depth to occupy the surface. In a second scenario he reasoned that if the charges had a mass then they would be required to occupy a depth of some small but finite thickness. He then argued that with the most sensitive instruments of his day no mass was ever detected when charges were placed on a surface, and, therefore, a zero thickness interface would be the best choice. Maxwell died in 1879 and it was not until almost 18 years after his death that J. J. Thomson discovered the electron had a mass and it was about

1000 times smaller that that of the hydrogen ion. The question which remains today is would Maxwell have changed his definition of surface charge density had he been privy to Thomson's information? Maxwell had also argued that Ohm's Law would be of no scientific value if it were not for the fact that the electrical conductivity were a single constant throughout the material (see [21], pp. 362-363). As a result, the zero thickness interface fit nicely with Ohm's Law.

7) Maxwell's Other Choice:

Maxwell's other choice for surface charge density was to define the surface itself as having some depth. One problem with defining a surface as having depth is that if charges suddenly reside within the surface then within that depth Ohm's Law will not apply. No one wants to get rid of Ohm's Law as it is a cornerstone of electrical engineering. However, it is clear that most of the problems that still remain in electrostatics involve the surface to some extent. Tribocharging is one area of poor understanding, but another area is that of the electrical discharge. In the Townsend discharge model (see [13], pp. 292-301) the number of ions grows through ionizing collisions in an electric field and this growth can be predicted by laboratory measurements of the first ionization coefficient. However, for an arc to occur these ions on reaching a surface generate – through collisions at the surface– counter charges which travel back in the direction opposite the incoming ions. Although the growth of an avalanche is well documented by Townsend's first ionization coefficient, the second ionization coefficient where the avalanche hits a surface is poorly understood and data on the second ionization coefficient are scattered an inconsistent [23] just like the data in the triboelectric series.

8) A realistic compromise:

Clearly a present day dilemma exists. On the one hand –in order to use Ohm's Law– the ZTI postulate must be kept which gives rise to a zero thickness interface. For an interface of zero thickness there is no known physics that would give rise to a method of tribocharging the surface. On the other hand, as the triboelectric series indicates, tribocharging appears to be material dependent so there must be some connection to some property of both materials at the interface which suggests the interface might have some depth in both materials.

One possibility of bridging this dilemma is to maintain the appearance of Ohm's Law while allowing an interface to have depth. As will be shown, this can be done by using the more general form of the charge flux equation (1) rather than the more restrictive charge transport equation (8) known as Ohm's Law. The main advantage of (1) is it can handle a $\nabla \rho_e$ when an interface is allowed to have a finite thickness. Furthermore, even when –as will be shown– there can be an internally produced E-field across an interface which can give rise to tribocharging, if an external E-field far exceeds any interface E-field, so that $\sigma \mathbf{E} \gg -D\nabla \rho_e - G \rho_e \nabla T$, then for that condition (1) still essentially reduces to (8) and all calculations can be done using Ohm's Law. Thus, as will be shown, (1) allows the study of the interface and gives a physical model for tribocharging yet reduces to (8) when a sufficiently large external E-field is applied; so, Ohm's Law remains intact for situations where the surface is not important.

C. Charge Flux Across An Interface

Using (1) it is possible to understand what occurs across an interface of finite thickness. Bring two solids together as depicted in Fig. 1 and examine the consequences of (1). For the sake of the discussion let solid material A have a higher free-charge number density n_{0eA} than solid material B so that $n_{0eA} > n_{0eB}$ which using (2) also means $\rho_{0eA} > \rho_{0eB}$. Since charges end up on the contacting surfaces when the solids are separated it is important to determine the reason why this happens. When the solids are brought together as depicted in Fig. 1, there must be at least some transient force causing the free-charges to move across the *yz* contacting plane at $x = 0$. No reference to the ZTI postulate is made here. When the contacting surfaces are together at a constant temperature T two things are known. There are no thermal gradients $\nabla T = 0$ and after any initial transient there can be no current **J** going across the interface or anywhere else in the steady state, so $\mathbf{J} = 0$ everywhere in the equilibrium state. Consider now the interface region of finite but as yet undefined thickness δ_{IF} as depicted in Fig. 1. In this region in the steady state and at a constant temperature when the two solids are in contact (1) reduces to

$$\mathbf{J} = \mathbf{J_e} = 0 = \sigma\,\mathbf{E} - D\nabla\rho_e.$$
(Electron Charge Flux Across An Interface) (9)

But (9) is composed of two charge flux terms

$$\mathbf{J_D} = -D\nabla\rho_e,$$
(Diffusional Charge Flux Across An Interface) (10)

and, since, $\mathbf{E} = -\nabla V$

$$\mathbf{J_E} = \sigma\,\mathbf{E} = -\sigma\nabla V.$$
(E-Field Charge Flux Across An Interface) (11)

So (9) indicates in the steady state it is possible to have two current densities –a diffusional current density (10) and an E-field induced current density (11)– flowing across an interface as long as the two current densities are in opposite directions and cancel each other out everywhere across the interface, namely $\mathbf{J_D} = -\mathbf{J_E}$ everywhere across the interface.

1) The Meaning of $\nabla\rho_e$

The diffusion charge flux (10) contains the gradient term $\nabla\rho_e$ and it is very important to understand just what this term represents at the interface. A free-electron is an electron that can be moved under the action of an applied force. In this paper it has been assumed that there are some free-electrons in all materials, an assumption consistent with other models. For example, in the Drude model for a metal valence electrons are assumed to be completely detached from their positive ions and form a free-electron gas. It is further assumed that there is a free-electron associated with each atom of the metal and the number density n_e of the free-electrons is equal to the number density of the ions n_+ (or atoms) and the charge density (2) of the free-electrons is therefore quite large. In a semiconductor the dopant atoms are assumed to each have a free-electron, and the number density of the free-electrons is equal to the number density of the dopant atoms. In an insulator a hopping mechanism is usually assumed to allow a few electrons to move, so the number density of the free-electrons is very low. The net result is even an insulator can be thought of as having

free-electrons although due to its low number density n_e and hence low charge density (2) its electrical conductivity (3) is, in some materials, so low as to not be measurable. The net result is that each material has a defined charge density (2) of free-electrons that can be moved when subjected to the action of a force.

When two different solids –each having its own value of charge density (2) of free-electrons– are brought together to create an interface, there is a sudden jump (at the $x = 0$ contact plane) in the magnitude of the charge density (2) of the free-electrons. In other words, there is a gradient $\nabla\rho_e$ of free charge density across the interface at $x = 0$. Through collisions this gradient causes the solid that has the higher free-electron charge density to move some of its free-electrons to the region of lower charge density which is in the other solid. This gradient $\nabla\rho_e$ creates the diffusional current density (10) and as noted in (10) the minus sign insures that the movement of the charge is in the opposite direction to the increasing charge density. That is to say the flow is in the direction opposite a positive gradient $\nabla\rho_e$.

2) The meaning of E or $-\nabla V$

The E-field charge flux (11) contains the electric field **E**, or since $\mathbf{E} = -\nabla V$, the gradient term ∇V, and it is also very important to understand just what this term represents within the interface region. Previously the diffusion term with gradient $\nabla\rho_e$ was discussed, and it was found –when two solids are together making contact– the material with the higher free-electron charge density (2) had a driving force through collisions to move some of its free-charges across the contacting area and into the other solid. As soon as this higher free charge density solid gives up some free-electrons, the region around its interface is no longer net neutral and becomes charged positive. Likewise, the interface region of the other solid which has accepted the free-electrons becomes charged negative. But any electric field **E**, by definition, starts on positive charge and ends on negative charge. Hence, in the interface region an electric field **E** begins setting up and this field begins to oppose the diffusion of free-electron charges. Since $\mathbf{E} = -\nabla V$ there is a potential difference that occurs across the interface region. The injected free-charges continue to spread out into the low-free-charge-density region, but $\nabla\rho_e$ becomes smaller the further these charges move from the contact interface. Eventually, as the charges move further away $\nabla\rho_e$ becomes very small $\nabla\rho_e \to 0$ after which the free charge density becomes indistinguishable from the solid's intrinsic free charge density. How far the charges move to reach this location defines the thickness of the interface within the lower-free-charge-density material.

A similar argument can be made in the higher-free-charge-density solid. It gives up free-electrons but the further back from the interface the smaller is the $\nabla\rho_e$ which (when viewed in the -*x* direction) eventually becomes very small $\nabla\rho_e \to 0$ beyond which the free-charge-density becomes indistinguishable from its intrinsic higher-free-charge-density.

3) Definition: Thickness of Interface:

Consider two materials in contact as depicted in Fig. 1 with solid material A having a higher free-electron concentration than material B, i.e., $n_{0eA} > n_{0eB}$. Further assume both materials

are of infinite extent in the *yz* plane so that the non-contacting surfaces (and any of their effects) can be ignored. For these conditions Fig. 1 can be described as having the space to the left of $x = 0$ filled with material A and having the higher-free-charge density ρ_e {see (2)} where $\rho_e = \rho_{0eA}$ everywhere when material A is isolated and where $\rho_e = \rho_{0eA}$ but only for all points some distance away from the contact surface when material A is in contact with material B. Likewise to the right of $x = 0$ material B will have $\rho_e = \rho_{0eB}$ everywhere when material B is isolated and where $\rho_e = \rho_{0eB}$ but only for all points some distance from the contact surface when material B is in contact with material A. When moving from $x \ll 0$ deep inside material A, then at a far distance in the -*x* direction $\rho_e = \rho_{0eA}$ but as ρ_e moves into the δ_{IF} region in Fig. 1 it is no longer equal to ρ_{0eA} and must transition across δ_{IF} to eventually have the lower value ρ_{0eB} in material B for all *x* beyond δ_{IF}. The transitioning from the intrinsic higher-free-charge density value in material A to the intrinsic lower value in material B is the distance defined as the thickness of the interface.

4) Calculating ρ_{0e}

In the region far away from the interface the material is ohmic and the electron-charge-density ρ_{0e} for both materials in contact can be calculated (although for insulators its value can be very hard to estimate). For a metal ρ_{0e} is relatively easy to calculate [[24]] (remember $\rho_{0+} = -\rho_{0e}$). For example, an elemental metal has a atomic weight A_w and has N atoms per unit cell of volume V_C and there are Avogadro's number $N_A \approx 6.022 \times 10^{23}$ atoms/mol; so, it has a mass density $\rho_m = (N A_w)/(V_C N_A) = n_C A_w / N_A$ where n_C is the number density of atoms in a unit cell. But n_C is also the same as the general number density of atoms in a homogeneous and isotropic material. Restated this equation gives $n_C = \rho_m N_A / A_w$. If some fraction f_{ie} of these atoms can be consider as fixed-position ions each with a free-charge then the number of free-charges per unit volume is $n_{0e} = f_{ie} n_C$, where $f_{ie} = 1$ in the Drude model, but experimentally, at low temperature, it has been found to have a value less than unity for the alkali [25] and alkaline [26] metals. Using (2) and defining ρ_{0e} as the intrinsic free-electron-charge-density far away from the interface then $\rho_{0e} = s_e q_0 n_{0e} = s_e q_0 f_{ie} n_C = s_e q_0 f_{ie} \rho_m N_A / A_w$.

For a semiconductor $n_{0e} = f_{ie} n_C$ is known from its manufacture since the impurity dopant concentration n_d replaces n_C and $f_{ie} = 1$, so, for a semiconductor $\rho_{0e} = s_e q_0 n_{0e} = s_e q_0 f_{ie} n_d = s_e q_0 n_d$.

For a good insulator the intrinsic free-charge-density is much more difficult to obtain. Values for the electrical resistivity of some solid insulator materials are given in [27]. The resistivity is the reciprocal of the electrical conductivity σ but assuming the conductivity is due to free-electrons still requires an estimate of the electron mobility b_e in the insulator in order to obtain ρ_{0e} {Note: away from the finite thickness interface $\rho_e = \rho_{0e}$ see (3)}. More discussion on insulators will be given later in section IV. Discussion {see subsection C. Thickness of the interface}.

5) Potential across an interface:

With $\mathbf{E} = -\nabla V$ and substituting the appropriate term for σ from (3) and for D from (5) gives (9) as

$$s \rho_e b_e \nabla V = \frac{-kT b_e}{q_0} \nabla \rho_e. \tag{12}$$

(Electron Charge Flux Across An Interface)

For two materials with their interface along the *yz* plane as shown in Fig. 1 (12) reduces further to

$$\rho_e d V = -\frac{kT}{s q_0} d \rho_e. \tag{13}$$

(For gradients only in x-direction)

But (13) can be integrated from far away from the surface in either material where the free-electron charge density is ρ_{0e} and the potential is V_0 (Note V_0 can be defined as the zero of potential for one of the materials) to some general point along *x* where charge density is ρ_e and potential is V which gives

$$\rho_e = \rho_{0e} \, e^{\frac{-s q_0}{kT}(V - V_0)} \tag{14}$$

On the other hand if (13) is integrated from far away from the surface in material A where the potential is V_{0A} all the way up to the contact surface at $x = 0$ where $\rho_{eA} = \rho_{eIF}$ and the potential is V_{IF}, and then, from $x = 0$ to far away from the surface in material B where the potential is V_{0B} the result –after recognizing it does not matter from which direction the integration is performed because when at $x = 0$ then $\rho_e = \rho_{eIF}$ and the potential is V_{IF}– is {see [28] for details}

$$V_{0A} - V_{0B} = -\frac{kT}{s_e q_0} \ln \frac{\rho_{0eA}}{\rho_{0eB}} = -\frac{kT}{s_e q_0} \ln \frac{n_{0eA}}{n_{0eB}} \tag{15}$$

(Potential Drop Across the Full Interface)

which is the potential drop across the full interface of thickness δ_{IF}. It is clear from (15) that the potential drop across the full interface is dependent on both solids –namely the free-electron charge (or number) densities of both materials. At a fixed temperature *T* the ln term in (15) determines the magnitude of the potential drop. For two metals in contact $n_{0eA}/n_{0eB} = (f_{ieA}/f_{ieB})(\rho_{mA}/\rho_{mB})(A_{wB}/A_{wA})$. Constantan is a copper-nickel alloy consisting of 55% copper and 45% nickel which, at this mixture, gives a calculated atomic weight of 61.27 g/mol and density of 8.9146 g/cc. Copper has a density of 8.92 g/cc and an atomic weight of 63.546 g/mol. A copper-constantan junction is a type T thermocouple. Assuming the Drude model for metals ($f_{ie} = 1$) and using (15) to calculate the potential of a thermocouple at 21°C with its cold junction at 20°C (so as to minimize any thermal expansion errors) gives only -3.1 µV/°C whereas the data for a Type T thermocouple shows +40 µV/°C. To obtain the correct sign and magnitude requires assuming constantan has (for the major component: 55% copper) $f_{ieCu} = 1$, and assuming only 12.5% of the nickel gives rise to free-electrons in the alloy state, so that $f_{ieCu} / f_{ieCon} \approx 1.65$. This would be consistent with the known electrical conductivity decrease for constantan $\sigma_{Const} = 2.04$ MS/m whereas $\sigma_{Cu} = 59.6$ MS/m and $\sigma_{Ni} = 14.3$ MS/m.

According to (3) this drop in σ_{Const} must be attributed to both decreases in free-charge density n_e and mobility b_e.

D. *Total charge density across an interface*

Up until now everything discussed concerned the free-electron charge density in either material. But what about the total charge density in the two materials? In the ohmic region away from the interface region there is both the free-electron charge density and the ion density of the atoms that make up the net-neutral total charge density in either of the two materials. It is only near the interface that the free-electrons try to diffuse across the interface. The total charge density $\rho = \rho_{0+} + \rho_e$ is the sum of the contributions from both the ions and the free electrons. Far away from the interface $\rho_e = \rho_{0e} = -\rho_{0+}$ and the material is neutral, i.e. $\rho = 0$. But everywhere the total charge density must satisfy Poisson's equation and for the situation in Fig. 1 (no variation in y and z) Poisson's equation –with the aid of (14)– is given by

$$\frac{d^2 V}{dx^2} = -\frac{\rho}{\epsilon} = -\frac{s_e q_0 n_{0+}}{\epsilon} - \frac{s_e q_0 n_{0e}}{\epsilon} e^{\frac{-s_e q_0}{kT}(V - V_0)} \tag{16}$$

The problem of studying the total charge density has been worked out in detail elsewhere [29] and only a few highlights are presented here to give some clarity to the concept of a thickness of the interface. For the solid-solid interface problem shown in Fig. 1, and after noting $n_{0+} = n_{0e}$ and $s_+ = -s_e$ and with the following change of the variable

$$Y = \frac{s_e q_0}{kT}(V - V_0) \tag{17}$$

Poisson's equation (16) in a solid material with permittivity ϵ reduces to

$$\frac{d^2 Y}{dx^2} = \kappa^2 (1 - e^{-Y}) \tag{18}$$

where

$$\lambda = \frac{1}{\kappa} = \sqrt{\frac{kT\epsilon}{s_e^2 q_0^2 n_{0e}}} = \sqrt{\frac{kT\epsilon}{q_0^2 n_{0e}}} \quad \text{(Debye Length)} \tag{19}$$

is the well known Debye length {see [19], p. 134}. At the present time no general, closed-form solution to (18) is known, but for the special case of potentials across the interface below a few millivolts, where $Y \ll 1$ (18) reduces to

$$\frac{d^2 Y}{dx^2} = \kappa^2 (1 - e^{-Y}) \approx \kappa^2 Y \tag{20}$$
$$\text{(valid only when } Y \ll 1; \text{ [LMA])}$$

which in [29] is referred to as the low-millivolts approximation (LMA), and (20) has the solution $Y = Y_s e^{\kappa x}$ for negative values of x (material A) and $Y = Y_s e^{-\kappa x}$ for positive x (material B). The boundary condition, at the interface located at $x = 0$, is $V = V_s$ where V_s is the potential at the contact surface; namely

$$Y_s = \frac{s_e q_0}{kT}(V_s - V_0). \tag{21}$$

Hence, for material A to the left of the origin, the solution to (20) gives the potential $V_A(x)$ along material A, and to the right it gives $V_B(x)$ along B as [29]

$$V_A(x) = V_{0A} + (V_s - V_{0A}) e^{\kappa_A x} \quad -\infty \leq x \leq 0^- \tag{22}$$

$$V_B(x) = V_{0B} + (V_s - V_{0B}) e^{-\kappa_B x} \quad 0^+ \leq x \leq \infty \tag{23}$$

In [29] the complete solution of the electrical behavior in both materials is presented and includes the electric fields, the potentials across the interface and the surface charge densities of the two materials before separation (Note: remember the surface region has a thickness so charges can and do reside within the interface). It will suffice here to mention that the surface charge densities contain the $\ln(n_{0eA}/n_{0eB})$ term found in (15) so their magnitudes are also controlled by the number of free-electrons in the two solid materials. For the sake of completeness it is noted that in [29] the surface charge densities are also affected by ϵ_A, ϵ_B and the temperature T.

IV. DISCUSSION

To understand tribocharging requires an understanding of what charges do when two solid materials contact each other to form a solid-to-solid-contacting-interface. In this paper it is assumed that all solid materials have some atoms each of which can be defined as consisting of a positive-charged ion and a free-electron. As a result any specific material will have a certain electron number density $n_e = n_{0e}$ and charge density $\rho_e = \rho_{0e}$ dispersed uniformly in the bulk of the material where n_{0e} and ρ_{0e} are constants and are properties of the specific material.

A. *Where the model succeeds*

In this paper the application of the charge flux equation (1) to the solid-solid interface has shown that the electron charge density ρ_e is subjected to a diffusional gradient at the solid-solid contact-interface. This results in a number of interesting implications for the interface and for tribocharging in general. First and foremost is that the interface has a thickness across which a potential exists. For small differences in the free-electron number densities between two solids that potential across the interface changes exponentially on either side of the contacting area of the surfaces as described by (22) and (23). The material's Debye length λ given in (19) is the controlling factor in the potential change, and the thickness of the interface within any material will be $\approx 5\lambda$. As a result, the total thickness of the interface will be $\delta_{IF} \approx 5\lambda_A + 5\lambda_B$. The full potential across the interface is given by (15) and can be quickly used to determine the sign of the charge on each material after tribocharging. Specifically, remembering that $-1/s_e = +1$ for electron transport, (15) reduces to simply $V_{0A} - V_{0B} = (kT/q_0) \ln(n_{0eA}/n_{0eB})$. But $\ln(1) = 0$ and $\ln(1^+) > 0$ whereas $\ln(1^-) < 0$, so when $n_{0eA} > n_{0eB}$ the ln term is positive and V_{0A} must be positive with respect to (wrt) material B. Alternatively, when $n_{0eA} < n_{0eB}$ the ln term is negative and V_{0A} must be negative wrt B. These results

explain the data of the *lower-half* of Table 1. Namely, the more conductive materials found in the middle of the table –have a higher electron number density n_{0e} and– are higher in the table and are always found to be positive when tribocharged after being in contact with a lower listed material. Likewise the lower listed material –has a lower electron number density n_{0e} and– is always found to be negative when tribocharged after being in contact with a higher listed material. Furthermore, the further the free-electron number density of an insulator is from that of a metal, the lower is the insulator in the table.

The model also explains the contact charging of hailstones with ice crystals in thunderclouds where the more conductive, impure hailstones become positive and the much less conductive pure ice crystals become negative [30].

B. *Where the model needs to be modified*

Although the above argument is in general agreement with the lower half of Table 1 the same argument fails miserably in attempting to explain the upper half of Table 1 because there the higher listed materials are positive even though the lower listed materials in the middle of the table have a higher electron number density n_{0e}. As a result, the theoretical analysis presented here is either wrong or something else is occurring that has not been described in the analysis.

It is this author's belief that the analysis presented here is correct and that assumptions 2, 6, 8 and 14 in Table 5 clearly state the theory is only applicable to two solids in intimate contact. The lower half of Table 1 usually satisfies the solid-solid contact requirement. In other words, it is the hydrophobic nature of observation 7 in Table 2 that allows the application of the theory presented in this paper. On the other hand, the upper half of Table 1 is ripe with solids which adsorb moisture and other hydrocarbon pollutants (see observation 6 in Table 2) and as a result an interfacial contact made in the upper half of a triboelectric series table is actually a solid-*adsorbed-liquid*-solid interfacial contact with the liquid layer intimately in contact with both the first and second solids. In the liquid layer the positive ions are free to move so (7) is no longer valid and Poisson's Equation as given by (16) is not valid in the liquid layer. As a result, the analysis presented in this paper is only valid for a solid-solid contact which is essentially the lower half of Table 1. For the upper half to be predicted by the charge flux equation will require a further analysis valid for a solid-liquid-solid interface.

The theory of a conducting liquid against a charged surface has been worked out and is known as the Gouy-Chapman theory [19] which gives a solution to Poisson's equation in the liquid. The theory predicts that a liquid in contact with a charged solid surface defined at a fixed potential has opposite charged ions in the liquid tightly bound to the surface in what is known as the Stern (or Helmholtz) layer, and adjacent to the Stern layer is a region called the diffuse layer containing loosely bound mobile ions. This Stern layer may be related to what is happening at the hydrophilic interface. As discussed in the next section the interface thickness of a liquid layer can be on the order of a micrometer whereas the actual thickness of the adsorption layer may only be on the order of a nanometer or less so all the physics of a solid-liquid-solid interface may be occurring within the Stern layer. For example consider an insulator in the upper half of Table 1 in contact with a metal in the middle of Table 1. If there is a liquid layer between the

insulator and the metal, then the higher free-charge density metal places electrons into the liquid and electron-attachment then creates negative ions in the liquid. In the liquid layer positive ions also exist and are free to move and will result in a layer of positive ions strongly attracted to the negative ions. But if the actual adsorption layer is only a couple of atomic layers thick these positive ions are also located at the insulator interface and become acceptors for free-electrons located in the insulator. Thus, the insulator gives up electrons making it positively charged. When the two materials are separated the liquid layer through coulomb attraction stays with the higher-charge-density metal leaving it negatively charged while the insulator having given up electrons remains positively charged. At the moment this is still conjecture as the physics of the liquid interface must be investigated using a charge flux equation for each specie that exists in the liquid in place of (7).

C. *Thickness of the interface*

Just how thick is an interface? The Debye length (19) for a metal like copper is $\lambda_{Cu} \approx 0.01 \, nm = 0.1 \, \mathring{A}$ so $5\lambda_{Cu} \approx 0.5 \, \mathring{A}$ suggesting only the outermost atomic layer of the metal takes part in supplying free-electrons for the diffusion. At the other extreme good insulators have very low conductivity as for examples PET ($\sigma \approx 10^{-21}$ S/m), Teflon ($\sigma \approx 10^{-24}$ S/m), Hard Rubber ($\sigma \approx 10^{-14}$ S/m), and most Glasses ($\sigma \approx 10^{-11}$ - 10^{-15} S/m). For polymers typical values of mobility are in the range of 10^{-15} $m^2V^{-1}s^{-1}$ to 10^{-8} $m^2V^{-1}s^{-1}$ {see [13], p. 211}. Compared to copper (n_{0e} = 8.5 x 10^{28} electrons/m^3; $\sigma \approx 6$ x 10^7 S/m) it can be seen using (3) that –assuming a typical mid-range insulator might have a conductivity of 10^{-10} S/m and an electron mobility of 10^{-11} $m^2V^{-1}s^{-1}$– a free-charge number density of a mid-range insulator is about $n_{0typ-insul}$ = 10^{20} electrons/m^3 {see [1], p. 26}. So the ratio of a metal to a mid-range insulator density is $\approx 10^8$ and ln $10^8 \approx 20$ which means at room temperature {20 ºC (293 K)} where $kT_{293} \approx 25$ mV the typical voltage across the interface is 500 mV, whereas for two metals at a full order of magnitude in density difference ln 10^1 = 2.3 and the typical voltage drop across the interface is at best only about 50 mV. Finally, a typical insulator will have a dielectric constant of $\epsilon_r = 3$; so, at room temperature it will have a Debye length of $\lambda \approx 205$ nm and $5\lambda \approx 1000$ nm = 1 μm as the depth of the interface for the typical mid-range insulator.

On the other hand, if a solid adsorbs moisture and it is in the form of pure water at a pH of 7 then $\lambda_{H_2O} \approx 1350 \, nm$. But water vapor is usually in equilibrium with the typical present day concentration (300 ppm) of CO_2 and has a pH around 5.6 $\left(\lambda_{H_2O} \approx 270 \, nm\right)$. According to the United States Environmental Protection Agency [31] as of the year 2000 the most acidic rain falling in the U.S. has a pH of about 4.3 giving $\lambda_{H_2O} \approx 60 \, nm$. In all these cases of moisture adsorption the Debye length is greater than the typical monolayer of adsorbed moisture so the adsorbed layer is fully involved at the interface. To reiterate this liquid adsorption problem has not been addressed in the model presented in this paper.

V. CONCLUSIONS

The charge flux equation was used to study the charging characteristics of a solid-solid contact interface. The simplified model chosen is one in which any solid material has some fraction of it atoms which can be treated as if each of these

atoms is a positive ion and a free-electron. The free-electrons are free to move under the action of any force including the force due to collisions. The charge flux equation showed that at the plane of interfacial contact the difference in the free-electron concentrations of the two contacting materials constituted a concentration gradient across the interface which caused a diffusion of free-electrons from the solid with the higher concentration to the solid with the lower concentration. Because the positive ions cannot move this free-electron diffusion leaves the material with the higher free-electron concentration positive while the material with the lower concentration becomes negative in the region of the interface. An E-field sets up within the interface region opposing the diffusion. In equilibrium the steady state of diffusion charge flux $\mathbf{J_D}$ is just offset by the E-field charge flux $\mathbf{J_E}$ so that the total flux $\mathbf{J} = \mathbf{J_D} + \mathbf{J_E} = 0$ everywhere in the two materials including in the interface region. The interface region depicted in Fig. 1 is found to be of a thickness $\delta_{IF} \approx 5\lambda_A + 5\lambda_B$ where from (19) $\lambda_A = \sqrt{kT\epsilon_A / \left(q_0^2 n_{0eA}\right)}$ an $\lambda_B = \sqrt{kT\epsilon_B / \left(q_0^2 n_{0eB}\right)}$ are the Debye lengths of the solid materials A and B respectively, ϵ_A and ϵ_B are the respective permittivities with n_{0eA} and n_{0eB} the respective concentrations of the free-electrons in the region away from the interface region of thickness δ_{IF}. The model has been used to successfully predict the features of the lower half of a generic triboelectric series table such as described in Table 1. The model does not predict the upper half of the table and that is believed due to the hydrophilic nature of the materials in the upper half of the table. Hydrophilic solid materials will have a double interface when in contact with another solid as it is no longer a solid-solid contact but rather a solid-*liquid*-solid contact. The liquid contact will require a further refinement to the analysis.

VI. NOMENCLATURE

Because the approach presented in this paper may be new to some who are engaged in the study of tribocharging, a table of nomenclature is presented in Table 6 to help the reader quickly grasp the terminology.

TABLE 6: NOMENCLATURE

Item	Description	Units
∇	Vector "Del" operator; $\nabla = \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}$	m⁻¹
$\hat{\mathbf{x}}$	Unit vector in the positive x direction	
$\hat{\mathbf{r}}$	Unit vector in the positive r direction	
ϵ_0	Permittivity of free space; $\epsilon_0 \approx 8.85$ x 10^{-12} F/m	F/m
ϵ_r	Dielectric constant of either solid (if the solids are A and B then the designation is ϵ_{rA} or ϵ_{rB})	
ϵ	Permittivity of either solid (ϵ_A or ϵ_B) Note: $\epsilon = \epsilon_0 \epsilon_r$	F/m
A	Cross sectional area; see Fig. 2	m²
A	Solid Material A	
A_w	Atomic weight of the solid	kg/mol
B	Solid Material B	
b_e	Electron mobility in the solid; see (4)	m²/(V·s)

Item	Description	Units
D	Electron diffusion coefficient in the solid; see (5)	m²/s
δ_{IF}	Total thickness of the interface; $\delta_{IF} \approx 5\lambda_A + 5\lambda_B$	m
E	Magnitude of the electric field	V/m
$\mathbf{E}$	Electric field vector $\mathbf{E} = E\hat{\mathbf{r}}$; and if only in x direction $\mathbf{E} = E\hat{\mathbf{x}}$	V/m
f_{ie}	Fraction of atoms in solid with free electrons; conductor $f_{ie} = n_{0e}/n_C$	
G	Electron thermophoresis coefficient; see (6)	m²/(s·K)
I	Magnitude of an electrical current; includes standard direction see Fig. 2	C/s
I_0	Absolute value of the current I; see Fig. 2 and section III.B.1) Ohm's Law:	C/s
$\mathbf{J}$	Total charge flux; for solid-solid contact this total is $\mathbf{J} = \sum \mathbf{J}_i = \mathbf{J}_+ + \mathbf{J}_-$ and $\mathbf{J}_+ = 0$	C/(s·m²)
$\mathbf{J}_+$	Charge flux of +ions; see (7)	C/(s·m²)
$\mathbf{J_D}$	Electron diffusional charge flux induced across the interface; see (10)	C/(s·m²)
$\mathbf{J_E}$	Electric field induced charge flux across the interface; see (11)	C/(s·m²)
$\mathbf{J}_e$, $\mathbf{J}_-$	Charge flux of free electrons (Note: $\mathbf{J}_e = \mathbf{J}_-$); see (1)	C/(s·m²)
k	Boltzmann's constant; $k \approx 1.38$ x 10^{-23} J/K	J/K
λ	Debye length; see (19); λ_A and λ_B are the lengths in solid A and in solid B	m
L	Length; see Fig. 2	m
ln(p)	Log to the base e; ln(e) = 1, see (15)	
m_e	Mass of the electron; $m_e \approx 9.11$ x 10^{-31} kg	kg
N	Number of atoms in a unit cell	
n_e	Electron number density anywhere including within δ_{IF} ; see (2)	m⁻³
n_{0+}	Ion number density anywhere in the solid; $n_{0+} = n_{0e}$ outside δ_{IF} region	m⁻³
n_{0e}	Electron number density outside the interface region of thickness δ_{IF}	m⁻³
N_A	Avogadro's number; $N_A \approx 6.022$ x 10^{23} atoms/mol	mol⁻¹
n_C	Number density of atoms in a unit cell; same as number density in solid	m⁻³
n_d	Semiconductor dopant concentration; typically $f_{ie} = n_{0e}/n_d$	m⁻³
n_e	Electron number density anywhere in the solid; n_e varies inside δ_{IF}	m⁻³
q_0	Charge on positron (or negative of charge on electron) $q_0 \approx +1.6$ x 10^{-19} C	C
q_e	Electron charge $q_e = sq_0 = s_e q_0$ $(s_e = -1)$	C
R	Resistance (resistor); see Fig. 2	W
ρ_+	Charge density of +ions anywhere in the solid; $\rho_+ = \rho_{0+}$ everywhere	C/m³
ρ_{0+}	Constant charge density of +ions in solid	C/m³
ρ_{0e}	Constant charge density of free electrons away from the interface region	C/m³

Item	Description	Units
ρ_e, ρ_-	Charge density of electrons anywhere in the solid; see (2) $\rho_e \neq \rho_{0e}$ within δ_{IF}	C/m^3
ρ_{eIF}	Volume charge density of electrons at the contact interface	C/m^3
ρ_m	Mass density of the solid	kg/m^3
σ	Electrical conductivity in the solid; see (3)	S/m
s	Sign of charge on a specie (for solids: $s = 1$ for +ions; $s = -1$ for electrons)	
s_e	Sign of charge on electron specie (for electrons $s =$	

Item	Description	Units
	$s_e = -1$)	
T	Temperature; $T(K) = 273.16 + T(°C)$	K
τ_e	Electron collision time; mean time between electron-atom collisions	s
V	Potential at x; $V = V(x)$	V
V_C	Volume of a unit cell	m^3
V_{IF}	Potential at the contact interface	V
Y	Dimensionless potential; see (17)	

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