Modeling of Small Gap Electrical Breakdown in Vacuum by Self-Heating of a Cathode Protrusion

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The present article focuses on the quasi-two dimensional modeling (1D 1/2) of the heat transfer through a cylindrical micro-protrusion on a titanium cathode, accounting for radiation loss and conduction heat as well as Joule and Nottingham heating. This self-consistent modeling provides temperature profile and voltage breakdown predictions depending on the macroscopic parameters (inter-electrodes distance, applied voltage drop, cathode material, etc.). Comparison with measurements of voltage breakdown taken from the literature shows fair good agreements and emphasizes the importance of the heat conduction properties of the considered cathode material and particularly of the protrusion geometry.

I. Introduction

Numerous applications such as X-ray tubes, electron microscopes, power vacuum switches, particle accelerators etc., use vacuum for the insulation of high voltages. Their performance reliability is limited by the risk of unpredictable breakdown event between electrodes. Moreover, breakdown leads to the formation of arc discharges, which can seriously damage the electrodes.

To better understand the origin of these unwelcome breakdowns, modeling is particularly interesting especially as arc dedicated tools have gained considerable advances due to many industrial applications (e.g. material processing, metallurgy chemical processes etc.). Thus, for the last decade, numerous simulations have been developed and reported, focusing mainly on issues such as plasma creation [1] and its expansion [2-6] or spot formation [7, 8] and spot motion [9-12]. All of them deal with the complexity of the arc discharge physics, which requires rather sophisticated numerical approaches.

From the technological point of view, pre-conditioning of electrodes is usually applied to decrease the frequency of the breakdown events. Hence a simplified numerical approach, which focuses on the correlation between the roughness of the electron emitter

electrode (cathode) and the macroscopic breakdown voltage (V_b), could lead to a better understanding of the pre-conditioning and its optimization. This is the idea of the present work which simulates the main elementary mechanisms occurring before the cathode-initiated breakdown (e.g. field and thermionic electron emission following the Murphy and Good's theory). The physical model basis and its numerical implementation are presented in Part II, whereas the simulation results are detailed and discussed in Part III, especially concerning the relevance of the obtained voltage breakdown/cathode surface roughness correlation.

II. NUMERICAL APPROACH

A. Physical model description

The model developed in this work assumes a cathode surface rather smooth, with a low density of defects, for which collective effects between nearby protrusions can be neglected. Let us consider this micro-protrusion as a cylinder of R-radius and H-height as characteristics of a Titanium cathode surface in a plane-to-plane discharge geometry, as shown in figure 1. The cathode surface is taken at T_p=300K and represents a thermostat regarding the thermal balance.

The protrusion under high applied voltage condition undergoes the main physical processes occurring in the thermo-emissive instability (i.e. radiation loss and conductive heat transfers, as well as Joule and Nottingham effects), which will lead to the plasma generation in the inter-electrode gap.

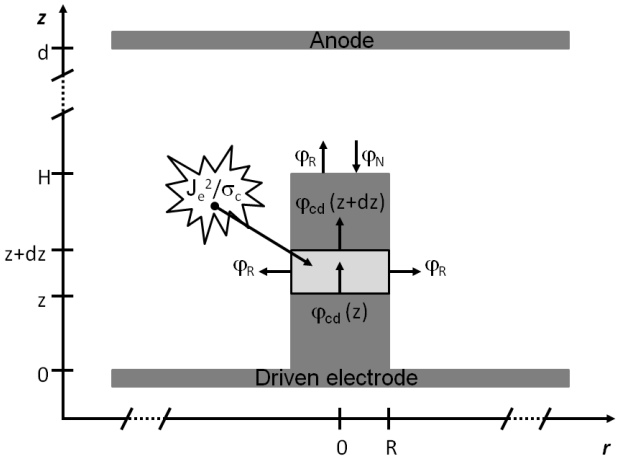

Fig. 1 : Scheme of the cylindrical protrusion at the cathode surface. – φ_{cd}, φ_R, φ_N, and J_e^2/σ_c represent the conductive, radiative, Nottingham surface fluxes, and the Joule volume heating.

Based on the heat transfer balance inside a slice of the micro-protrusion, the 1D heat equation for the micro-protrusion can be derived, assuming that the gradients along the

radial direction can be neglected comparing to the ones along the z direction.

The 1D heat equation is expressed as:

$$\lambda\pi R^2 \frac{d^2T}{dz^2} = -\pi R^2 \frac{J_e^2(E,T)}{\sigma_c} + 2\pi R\varepsilon_s\sigma_s\left(T^4 - T_\infty^4\right) \quad (1)$$

With the following boundary conditions:

$$-\lambda\left.\frac{dT}{dz}\right|_{z=H} = -\varphi_N(E,T) + \varepsilon_s\sigma_s\left(T^4 - T_\infty^4\right) \quad (2)$$

$$T(z=0) = T_P \quad (3)$$

Where φ_N represents the heat flux at the protrusion tip due to the Nottingham effect:

$$\varphi_N = -\frac{J_e(E,T)}{q_e} W_N(E,T) \quad (4)$$

σ_s is the Stefan constant. λ, σ_c, and ε_s represent the titanium thermal, electrical conductivities and the surface emissivity respectively. As λ and ε_s have only a small dependence on the temperature in the range of 300-2000 K (see Ref. [13-14]), averaged values have been considered:

$$\langle\lambda\rangle = 29.8 \quad \mathrm{W.m^{-1}.K^{-1}}$$

$$\langle\varepsilon_s\rangle = 0.55$$

However, the expression of [15] has been used for the electrical conductivity σ_c:

$$\sigma_c(T) = 5.76\times10^5 + 1.87\times10^6 \exp\left(\frac{T}{274.9}\right) + 1.37\times10^6 \exp\left(\frac{T}{274.9}\right) + 22.49\times10^6 \exp\left(\frac{T}{52.11}\right) \quad \mathrm{S.m^{-1}} \quad (5)$$

Following the Murphy and Good's theory, the electron current density (J_e) and the Nottingham energy (W_N, see Ref. [16-18]) can be written as analytical functions of the local temperature T(z) and the microscopic electric field E. J_e and W_N have been assessed using the Titanium values for the Fermi energy and the work function, which are 9.88 eV and 3.80 eV, respectively, according to [20].

The microscopic electric field E is expressed as:

$$E = \beta \cdot E_{macro} \quad (6)$$

where β is the field enhancement coefficient and E_{macro}, the macroscopic electric field.

In a plane-plane discharge geometry in vacuum and far from the electrodes edges, E_{macro} is obtained by dividing the applied voltage V_{app} by the inter-electrodes distance d.

β is assumed in our modeling to depend only on the protrusion geometry, especially on

its aspect factor F=H/R.

An analytical fit of β(F) has been derived over a wide range of aspect factor values, from 2D-axisymmetrical numerical solution of the Poisson equation performed with COMSOL Multiphysics® [21] for cylindrical protrusion geometries:

$$\begin{cases} \beta = 2 + 0.6F, \ for \ 0 \le F < 50 \\ \beta = 8 + 0.5F, \ for \ 50 \le F < 500 \\ \beta = 2600 - 2623 \times exp\left(-\frac{F}{4439}\right), \ for \ F \ge 500 \end{cases} \quad (9)$$

Let us underline that the radial characteristic dimension (R) affects not only the enhancement factor β through F (see Eq. (9)), but also the radiative and Nottingham fluxes as well as the conductive heating transfer (see Eqs. (1) and (2)). Even if the lateral gradients are neglected, the proposed model depends implicitly on R. Hence, the numerical approach is considered as 1D 1/2.

B. Numerical treatment

The numerical solution of the set of equations has been computed using the robust LU direct method for algebraic systems. Moreover, due to the high non linearity of the Joule heating and radiation loss terms, an iterative numerical scheme with over-relaxation of the temporary solution has been implemented in order to get the full convergence of the equation solution.

The obtained numerical results can either describe the protrusion stable steady-state with a maximum temperature below the fusion one, or represent the overheating (fusion) of the protrusion tip. The latter case is used to define the threshold value for the electrostatic field above which the discharge breakdown due to the thermo-emissive instability can occur.

III. RESULTS

A. Importance of the main physical processes on the micro-protrusion temperature profiles

The figure 2 shows three axial temperature profiles obtained for different numerical conditions. They differ from each other in the considered physical processes: test-case 1 – full calculation –, test-case 2 – without accounting for the Nottingham effect –, test-case 3 – without accounting for the Nottingham effect and the radiation loss –.

For all the three profiles, the following parameters have been chosen: V_{app}=78 kV (2 kV less than V_b), d=0.6 mm, T=300 K, β=40 and E=5.21×10^9 V/m.

The comparison between the test-cases 1 and 2 enlightens the importance of the Nottingham effect, which heats the surface in this electric field condition, whereas the similar

shapes of the profiles obtained for the test-cases 2 and 3 (5×10^{-4} % discrepancies) show the negligible impact of the radiation loss at the protrusion surface.

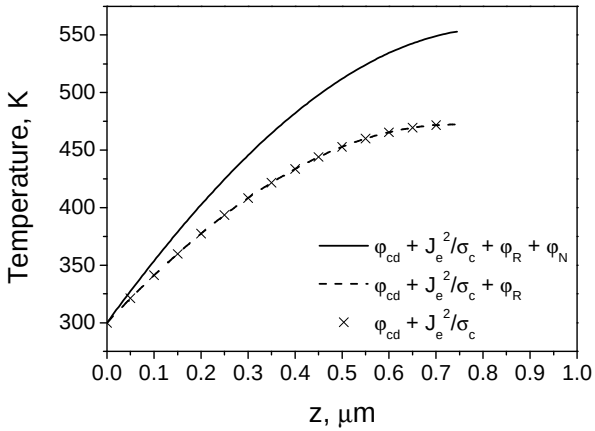

Fig. 2: Axial temperature profiles for three computation test-conditions.

Therefore, a first conclusion of our calculation is that for a 300 K-cathode temperature and for microscopic electric field values close to the critical one of about $4\text{-}6\times10^{9}$ V/m, the radiation loss can be safely neglected, the thermal balance of the micro-protrusion being essentially controlled by Joule heating, Nottingham effect and thermal conduction cooling. The small contribution of the radiation loss at the surface is also an indication that radial heat transport plays a rather negligible role. As a consequence, the radial profile of the temperature is nearly constant, which means that our 1D 1/2 approach is valid, at least, for Titanium and for the considered protrusion geometry.

B. Comparison with experiments

Using the thermo-emissive instability as the signature of the electrical breakdown, the code is able to predict the breakdown voltage V_b for a given inter-electrodes distance d.

These predictions have been compared to the experimental results of Beukema [22]. The Beukema's experiments consist in measurements of breakdown voltage for a pair of unconditioned titanium electrodes distant from d with $0.1 < d < 1$ mm. Beukema's results show a linear dependence of the breakdown voltage with d.

In fact, for such short inter-electrodes distances, the breakdown voltage is expected to vary linearly with d according to Latham [23] and breakdowns are considered to be initiated by electronic emission, process which is described by our code.

For $d > 2$ mm, other mechanisms could play a role such as breakdown initiation on macro-particles or micro-protrusion collective effects etc. In those conditions, $V_b(d)$ is expected to follow a power law.

By considering only short inter-electrodes distances in the novel developed code, the obtained good agreement with experimental data indicates that these other mechanisms

are negligible.

Beukema's measurements exhibit a constant field enhancement coefficient value of 40. In order to get a direct comparison with the experimental results, we have performed computations with protrusion aspect factors having β=40. Thus, the protrusion aspect factor considered in the modeling is representative of the average one on a realistic Titanium surface, justifying the same level of heating for all the cathode micro-protrusions simultaneously. Such a mechanism is considered to lead to the electrical breakdown between electrodes, as assumed in our model.

In the case of a cylindrical protrusion and using values R = 0.0117 μm and H= 0.75 μm, a very good agreement is obtained between our results and the experimental ones as shown in figure 3.

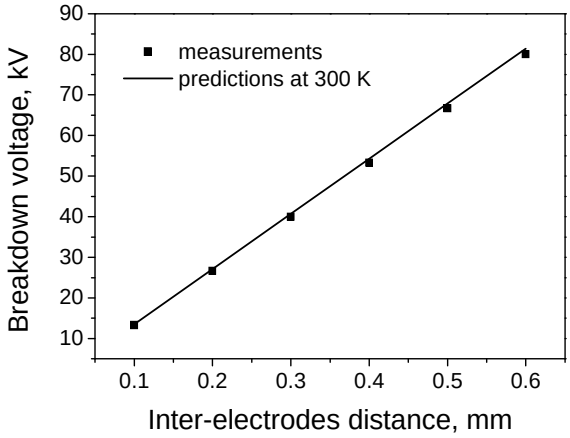

Fig. 3: Comparison between experimental measurements of [22] and our model predictions at 300K for the breakdown voltage as a function of the inter-electrodes distance.

C. Influence of the cylindrical protrusion geometry

Keeping the ratio H/R constant, it is also interesting to analyze the influence of the protrusion size on the breakdown voltage. The comparison of the corresponding breakdown voltage when multiplying by a factor of 8 both radius and height is presented in figure 4.

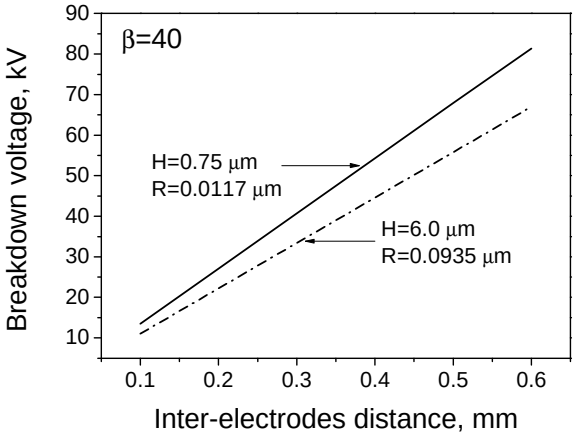

Fig. 4: Influence of the protrusion geometry on $V_b(d)$ assessment.

We can observe in Figure 4 a difference in between the slopes of the two curves. By increasing the protrusion size by a factor of 8, the breakdown electric field has been decreased by nearly 20%, passing from 5.44×10^{19} to 4.46×10^{19} V/m.

The figure 5 shows for a given inter-electrodes distance (i.e. d=0.6 mm) and β=40, the evolution of the breakdown voltage with the protrusion height. This evolution is proportional to $H^{-0.094}$.

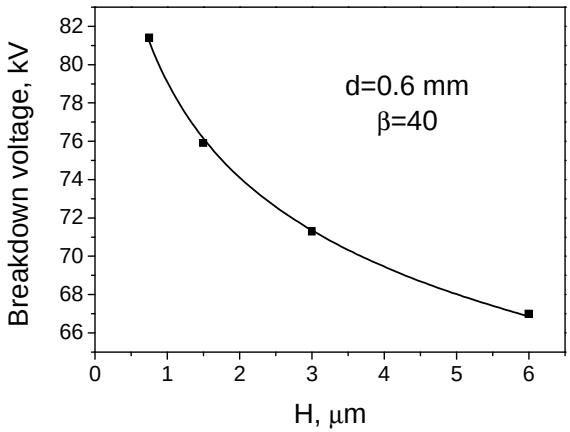

Fig. 5: Evolution of the breakdown voltage with the protrusion height for a constant value of β (β=40).

The results of figures 4 and 5 are in contradistinction with the Murphy and Good's theory, in which the protrusion geometry is only included through the β factor.

The reason of this is that for a given value of the microscopic electric field, Joule and Nottingham heating terms are identical, whereas the cooling through heat conduction will decreases with the length of the protrusion. Therefore, bigger size leads to higher temper-

ature and to lower breakdown voltage. It has to be noted that the contribution of the radiation loss, which depends on the protrusion surface/volume ratio will reinforce this tendency.

IV. Conclusions

We have developed a physical model able to predict the thermo-emissive instability appearance in a cylindrical micro-protrusion on a metallic electrode. Under the assumption that this instability leads to the electrical breakdown, our model can determine the breakdown voltage for given protrusion geometrical parameters at short inter-electrodes distance. Our calculations of breakdown voltage for Titanium are in good agreement with the experimental results of [22]. Concerning the protrusion geometrical effect on the breakdown voltage, our calculations demonstrate that the breakdown voltage depends not only on the field enhancement factor but also on the protrusion size, which, generally, is not known experimentally.

On a theoretical level, our main result lies in the emphasis of the necessity to account for the heat transfers inside the protrusion, mainly due to conduction, in order to correctly predict the breakdown voltage.

The present achievement can take this study to the new level towards the development of the macroscopic simulation of a realistic cathode surface roughness using Monte-Carlo approach in order to predict protrusion collective effects and voltage breakdown for large inter-electrodes distances (d > 2mm).

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