Migration of a sedimenting drop due to a tilted electric field

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Abstract

The influence of an electric field, tilted with respect to the vertical sedimenting direction, is studied with regard to the direction of drop sedimentation. It is observed that there is appreciable deflection of the drop in the horizontal direction due to the action of the tilted electric field. This is in contrast to the case of a horizontal applied electric field and a vertical electric field where the symmetries in the sedimenting direction do not lead to a net force in the lateral direction, which is perpendicular to the sedimenting direction. Experiments are performed with silicone oil (may be regarded as a dielectric oil) in hydraulic oil and aqueous NaCl solution (may be regarded as a conducting phase) in hydraulic oil. The direction of deflection observed in both the cases are opposite in nature. It is hoped that the experimental findings may help in designing gravity driven drop separators.

I. Introduction

Electrohydrodynamics (EHD) refers to the influence of electric fields on the motion of fluids and has been an area of research since the pioneering work of Zeleny [1], [2] and Taylor [3], [4], [5]. The presence of strong electric fields leads to the generation of an electric stress (known as Maxwell stress [6]) apart from the hydrodynamic stresses at the interface between two fluids. The initial studies regarded the media in such kinds of flows as insulating in nature[7], [8]. However, this consideration led to qualitatively different results pertaining to effects of high electric fields on the deformation of drops in the presence of electric fields. The discrepancy between experiments and theory was resolved by Taylor who put forth the theory of leaky dielectrics[5]. This entails the fact that even insulating fluids have a vanishinly small yet finite value of electric conductivity which results in altered distribution of surface charges as opposed to an analysis with perfectly insulating fluids. Since the proposition of the leaky dielectric model there has been a lot of work concerning the influence of electric fields at the interface between two fluids [9], [10].

The action of electric fields on interfaces was demonstrated by Taylor and McEwan[4], [11] theoretically and experimentally for the case of an interface between two fluids acted upon by DC voltages. The conclusion of their study was that the voltage necessary for the dielectric breakdown is significantly larger than the voltage required for deforming the interface via the action of normal Maxwell stresses overcoming the influence of surface tension at the interface[12], [13], [14]. The same analysis was then performed for the case of deformation of drops[5], [15]. Since then the area was pioneered by the works by Melcher et al. who studied the electrohydrodynamic phenomenon for interfaces, drops, and particles[16], [17]. The manipulation and separation of particles and drops has been investigated in recent times owing to its practical utility in many lab-on-a-chip applications [18]. A variety of physical factors influence the motion and deformation of particles and drops acted upon by an electric field. In the leaky dielectric model, the difference in the properties of the suspending mediaum and the drop leads to the an accumulation of a surface charge even in the case of seemingly insulating fluids. This surface charge may redistribute itself on the surface of the drop owing to the finite Reynolds number. The sweeping of charge due to the surface flow has been investigated and shown to cause significant variations in the transient and steady state deformation of drops and bubbles.

When a drop sediments in a medium in the absence of any other electric field, the drop achieves a terminal velocity termed as the Haramard-Rybczinksy velocity[19] which arises from the balance of the volumetric gravity force acting on the drop and the hydrodynamic drag acting upwards. Xu and Homsy[20] had shown through experiments and theory that the presence of a strong vertical electric field leads to a faster/slower sedimentation of a leaky dielectric drop in a leaky dielectric domain. It is also known that the influence of a horizontally applied electric field to a sedimenting drop only alters the sedimentation velocity. In both the cases, there is no lateral force acting on the drop. However, it was shown in an earlier work theoretically that the presence of a tilted electric field leads to a breaking of symmetry owing to which there is a net lateral force acting on the drop, thus resulting in a lateral motion [21], [22]. The purpose of this work is to experimentally show that the presence of a tilted electric field may be helpful to conceptualize a drop sorter based on the varying properties of the drop and continuous phase.

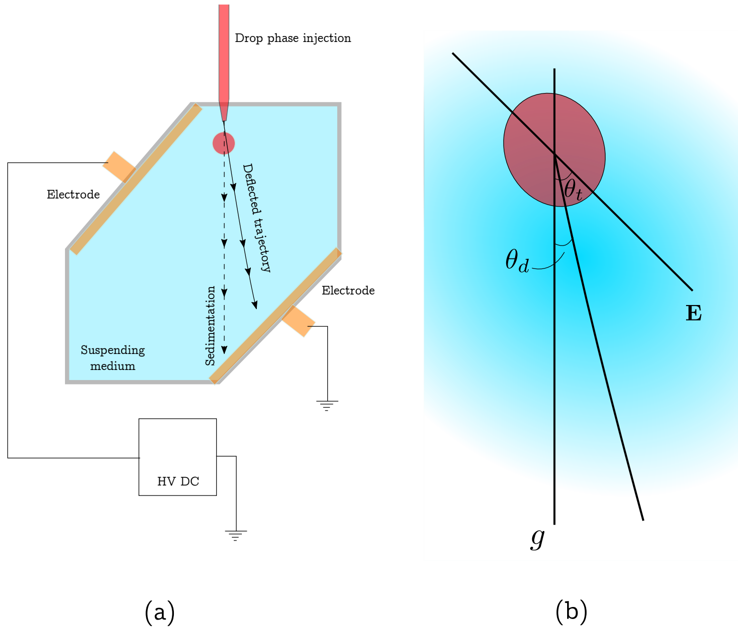

Fig. 1. (a) Schematic of the experimental setup. The cell consists of two electrodes as shown in the figure. The lower electrode is grounded and the top electrode is connected to a high voltage DC source. The drop is injected in the suspending medium from the top of the cell which is open to the atmosphere. The lighting is done from the back so that it is uniform and the viewing is done by means of a standard DSLR camera from the front (consequently, the actual view is as seen in the schematic itself). (b) Due to the action of the Maxwell stress because of the tilted electric field, the drop deforms and experiences a net lateral force. The angle the electric field makes with the vertical is termed as the tilt angle, θ_t while the angle made by the sedimenting drop with the vertical is termed as the deviation angle, θ_d.

II. Experimental Setup

The experimental setup consists of a cell made of plexiglass such that there are two electrodes attached to it at a 45 degree angle. The distance between the two electrodes is 10 cm. The dimensions are chosen so that the influence of the wall is minimized. The size of the rounded brass electrodes is 5 cm $\times$ 7 cm. The drop is injected at the top of the container. The high voltage source is indigeneously made by Zeal Mfg and Calibration services Pvt. Ltd. (Pune, India) and is able to output a maximum of 10 kV DC. The cell is filled with hydraulic oil, SAE-30 (brand name Relstar, Reliance, India). In the oil filled setup, the electric field is switched on and drops are injected manually using a syringe. The recording is made from the front viewing face by means of a DSLR camera (Nikon D5600) and the standard 18-55 mm lens. The images are processed in Matlab using functions available from the image processing toolbox. Experiments are performed with 1 M NaCl (Sigma Aldrich) solution and 100 cSt grade silicone oil (Sigma Aldrich). The various physical parameters are tabulated below:

Fluid name	ρ (kg/m^3)	μ(mPa·s)	σ (S/m)	ϵ_r
SAE-30	878.4	183.9	5.23×10^{-8}	4
Silicone oil	965	0.1	10^{-12}	2.8
Aqueous NaCl	996	0.89	8.5	80

TABLE I
Physical properties of the fluids employed in the present study.

III. Results and Discussion

In figure 2 an overlay photograph of the process of sedimentation in the absence and presence of an electric field is depicted. Subfigure (a) depicts the case for aqueous NaCl (1M) while (b) represents the case for silicone oil

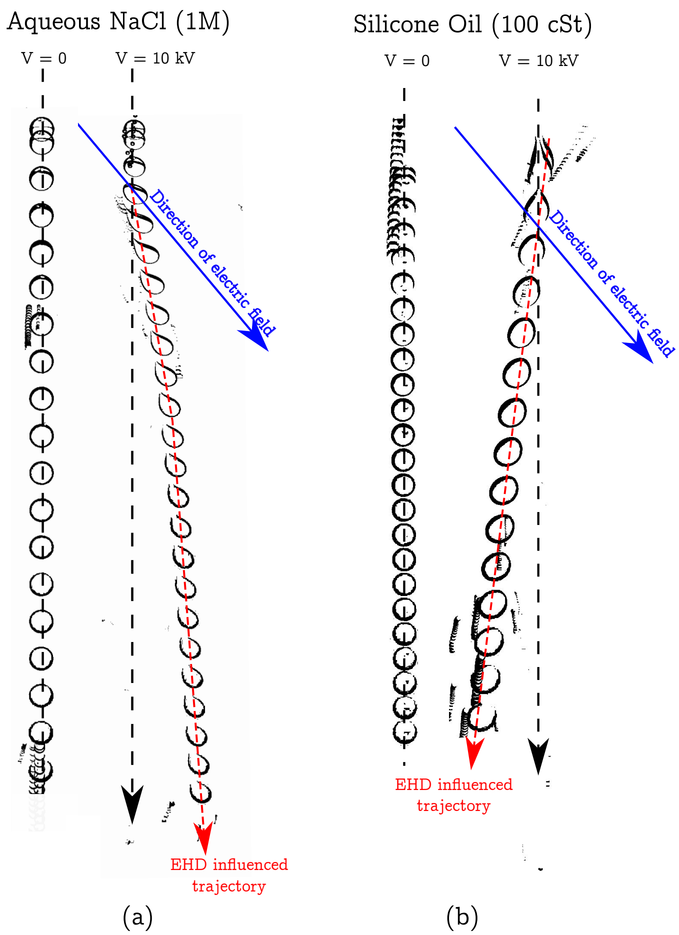

Fig. 2. Photographs depicting the deflection of a drop of (a) Aqueous NaCl (1M) solution and (b) Silicone Oil (100 cSt). The left trajectory in the two subfigures depicts pure sedimentation corresponding to the case where there is no applied electric field. The right trajectory corresponds to the situation where the applied voltage is 10 kV. A change in the direction of deflection is obtained for the two different drop phases owing to their markedly different conductivity and permittivity.

(100 cSt). It is observed that for both the dispersed phases there is no deviation from the vertical direction for pure sedimentation, i.e. when there is no applied electric field. Due to the action of the electric field the drop deforms. The deformed drop shape is clearly seen in the figure. The drop of NaCl deforms into an ellipsoidal shape with the major axis aligned with the direction of the electric field. On the contrary, the drop of Silicone oil deforms in into an ellipsoid whose minor axis is aligned with the electric field. This qualitative difference in the two physical quantities is what leads to the fundamental difference in the nature of deflection with the vertical.

The reason for the observations in the deformation may be understood by considering the physical properties of the two fluids relative to the continuous medium. Relative to SAE-30 oil, aqueous NaCl has a drastically high conductivity. The relative ratios of the conductivity, which can be denoted as $R = \sigma_d/\sigma_c$ (where σ_d and σ_c represent the conductivity of the drop and continuous medium respectively), is $R = 1.6252 \times 10^8$ while the relative ratios of the permittivity is $S = 20$. For silicone oil, the ratios are obtained as $R = 1.9120 \times 10^{-5}$ and $S = 0.7$. In the first case $R \gg S$ while in the second case $R \ll S$. If a static drop is considered without being suspended in the same fluid, the former case implies that the charge density is such that the side facing the positive electrode attains a negative charge. The charge distribution for a drop placed in an electric field point along the z (gravity) direction is given by

$$3E_\infty P_1(\cos\theta)\epsilon_c\epsilon_0 \left(\frac{R-S}{R+2}\right), \tag{1}$$

where E represents the magnitude of the far field velocity, $P_1(\eta) = \eta$ represents the first order Legendre polynomial, and ϵ_c and ϵ_0 represents the relative permittivity of the continuous medium and permittivity of vacuum respectively.

The charge distribution is typical of the charge distribution around a conductor imposed by a uniform electric field far from the domain. In this case the drop is expected to stretch into an ellipsoidal shape so that the major axis is aligned with the direction of the electric field. On the contrary, if for an insulating body in a domain with a far field uniform electric field, the charge distribution is such that positive charges (lesser in magnitude) are attracted towards the positive electrode. In this case the drop deforms in the opposite sense to that of a conductor. The situation well describes the deformation. However, in figure 2 it is seen that the bottom portion is flatter than the top. This is attributed to the influence of charge convection which is caused by the local sedimenting velocity causing the charges accumulated towards the bottom of the drop to be convected upwards.

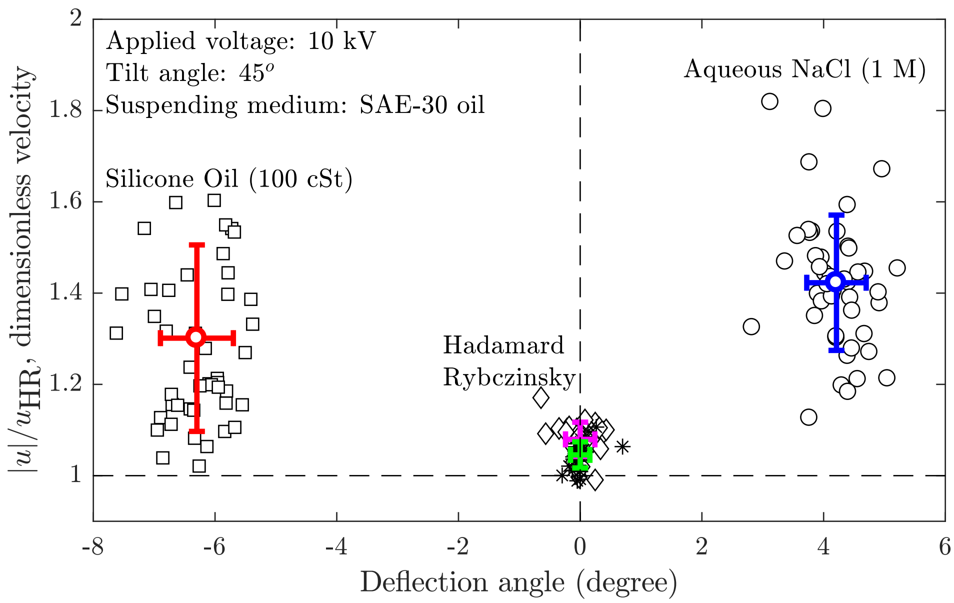

Fig. 3. Dimensionless velocity plotted against the deflection angle of a drop as it sediments through the medium (SAE-30 oil). The deflection in the direction of the electric field occurs for the case of Aqueous NaCl solution while the opposite deflection occurs for the case of silicone oil. The deviation from the Hadamard Rybczinksy velocity denoted by u/U_{HR} shows that apart from deviation from the vertical direction, there is also a speeding up of the settling process. The diamond marker and asterisk marker represent the settling process of silicone oil and aqueous NaCl respectively in the absence of any applied voltage.

In figure 3 the settling velocity and the deflection angle for the two fluids for both the cases of an applied voltage equal to 0 and 10 kV respectively are depivted. It is seen that there is a distinct correlation between the magnitude of the settling velocity and the deflection angle for the two fluids. When the velocity is non dimensionalized against the Hadamard Rybczinksy velocity,

$$u_{\mathrm{HR}} = \frac{2}{3} r^2 g \frac{\rho_d - \rho_c}{\mu_c} \frac{\mu_c + \mu_d}{2\mu_c + 3\mu_d}, \tag{2}$$

where g represents the acceleration due to gravity, ρ_c and ρ_d represent the densities of the continuous and drop phase respectively. Thus the role of the electric field in assisting the settling motion of the drops is assessed.

IV. Conclusion

In this study the deflection of a sedimenting drop from the vertical direction due to the action of a tilted electric field is studied. Depending on the nature of the drop and medium the drop may either elongate into an ellipsoid in the direction of the applied tilted electric field or in the opposite direction of the applied electric field. Consequently, the shape deformation leads to an alteration in the sedimentation process as manifested through the deflection from the vertical direction. It is hoped that the conclusions drawn from this work may help in the design of sedimentation based fluid-fluid separators.

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