Flow electrification in turbulent flows of liquids - Comparison of two models

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Abstract— **This paper deals with flow electrification phenomenon for liquids flowing through pipes of circular cross section in the case of turbulent flows. Two models are proposed. One based on the comparison of the magnitude of the radial fluctuant velocity with the magnitude of the relaxation velocity of the diffuse layer. Then, the zone in which the fluctuant velocity is greater than the relaxation velocity, the diffuse layer is fully perturbed and the charge is homogenized, while when the relaxation velocity is greater than the fluctuant velocity, the diffuse layer is considered not at all perturbed. The charge transported by the flow is then calculated with this new charge profile. The second model consists to compute the eddy diffusivity of the flow in terms of the radial coordinate of the pipe and to solve the system of the diffuse layer equations perturbed by the turbulence taking into account the eddy diffusivity effect; then with the profile of the charge obtained in the cross section the charge transported is calculated. Finally, the predictions of the charge transported using each model are compared and we discuss the performances of each.**

I. INTRODUCTION

The phenomenon of flow electrification has been investigated for more than fifty years now [1-5]. However, due probably to the complexity of the phenomenon, the influence of the turbulence has not been a lot investigated [6-10]. In fact two theories have been used, one based on the comparison of the fluctuant radial velocity of the flow with the relaxation velocity of the diffuse layer, the other one introduce the effect of the eddy diffusivity in the equations of the diffuse layer. Both methods need complicated calculus often solved numerically, even quasi totally in the case of the second method. Thus, with the rapid increasing of the performance of the computers in the last decades it has been possible to perform these calculi more easily with a better approximation. This is the goal of this research. In the first part of the paper we will describe the different hypothesis used for each theory. Then we will examine the fluid mechanic characteristics used in the case of the first theoretical model (fluctuant radial velocity versus diffuse layer relaxation velocity). As well we will examine the fluid mechanic characteristics used in the case of the second theoretical model (in terms of eddy diffusivity). Then the equations of the diffuse layer and the effect due to the turbulence are examined with its influence on the charge in the diffuse layer. Finally we present the results obtained for the space charge transported by flow electrification considering each model.

II. Hypothesis used and results obtained using each models

At first two hypotheses are available for both models: we suppose that the space charge density on the wall of the pipe is constant whatever the flow (laminar or turbulent) and for any Reynolds number. This hypothesis stems from the corroding model established previously [11]. The second hypothesis is, in fact a simplification, we consider that the ions coefficients of diffusion is the same for cations and anions. In fact, the ions responsible for the flow electrification phenomenon are often due to the dissociation of impurities which are not known, thus it is difficult to define an exact value of category of ions. Thus by simplicity we consider that both categories have the same diffusion coefficient D0.

A. First model

1) Electrical quantities

In this model we used the equations of the diffuse layer existing in a pipe of circular cross section for a liquid at rest or flowing with a laminar flow. Indeed, it is known that a laminar flow does not perturb the profile of the diffuse layer, thus the profile of the space charge density in a laminar flow is identical to that in a liquid at rest.
So, we consider the general non dimensional equations for a diffuse layer at rest:

$$\begin{cases} \overrightarrow{grad_+}\rho_+ + \sqrt{\rho_+^2+1}\ \overrightarrow{grad_+}\psi_+ = 0 \\ \Delta_+\psi_+ = -\rho_+ \end{cases} \tag{1}$$

Where ρ_+ is the non dimensional space charge density and ψ_+ the non dimensional potential.

In the case of a pipe of circular cross section, these equations become:

$$\begin{cases} \dfrac{d\rho_+}{dr_+} + \sqrt{\rho_+^2+1}\ \dfrac{d\psi_+}{dr_+} = 0 \\ \dfrac{d^2\psi_+}{dr_+^2} + \dfrac{1}{r_+}\dfrac{d\psi_+}{dr_+} = -\rho_+ \end{cases} \tag{2}$$

The first model makes an additional assumption concerning the total space charge in the diffuse layer. It is supposed that the total space charge in a whole section remains constant. This assumption is based on the hypothesis that the charge very close to the pipe wall is not affected by the turbulence, therefore no additional current would appear when the charge in the diffuse layer is homogenized in the central part of the pipe.
From equation (2) three zones can be distinguished:
- when $\rho_+^2 << 1$, in practice for $\rho_+ < 10^{-2}$, ρ_+^2 is negligible compared to 1, we have an analytical solution in terms of modified Bessel function. We call this zone the weak space

charge density zone.
- when $\rho_+^2 >> 1$, in practice for $\rho_+ > 10^2$, 1 is negligible compared to ρ_+^2, we have another analytical solution. We call this zone the strong space charge density zone.
- when $10^{-2} < \rho_+ < 10^2$ no simplifications are possible all the computation must be numeric. We call this zone the middle space charge density zone which exists in many experimental cases.

Practically, depending on the non dimensional radius of the pipe R_+ we can have only one zone for very small radii and two, or even three, different zones for greater radii. One example is plotted in figure 1. In this example the non dimensional radius of the pipe is equal to 1, the non dimensional space charge density on the center is equal to 3.5 and the non dimensional space charge density on the wall of the pipe is equal to 11.3. Thus this example is a case of middle space charge density in the whole section.

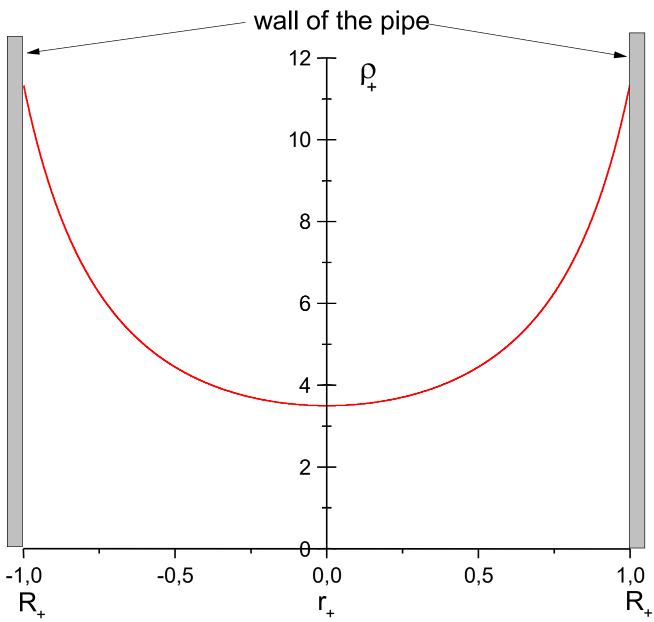

Fig. 1. Example of the evolution of the space charge density for a diffuse layer at rest or in a laminar flow

2) Flow characteristics

In a turbulent flow we introduce the friction velocity U* [12] and the non dimensional quantity η:

$$\eta = \frac{yU^*}{\nu} \tag{3}$$

With y=R-r (the distance to the pipe wall) and ν being the kinematic viscosity of the liquid flowing. To express the evolution of U/U* in terms of η we define three regions:

- The sub laminar region in which U/U*= η (this region corresponds to low Reynolds numbers Re)

- The universal velocity distribution laws for very large Reynolds numbers:

$$\frac{U}{U^*} = 2.5Log(\eta + 2.2) \tag{4}$$

- Between these two regions exist one call generally the transition region in which we have introduced an analytical law:

$$\frac{U}{U^*} = 6.624\tanh(Log(\eta) - 2.533) + 9.3 \tag{5}$$

We can see in Fig.2 the evolution of U/U* in the three regions. η_1 being the limit between the sub laminar region with the transition region and η_2 being the limit between the transition region and the universal law region for large Reynolds numbers. These theoretical laws are then compared to experiments made by Nikuradse and Reichard in Fig.3.

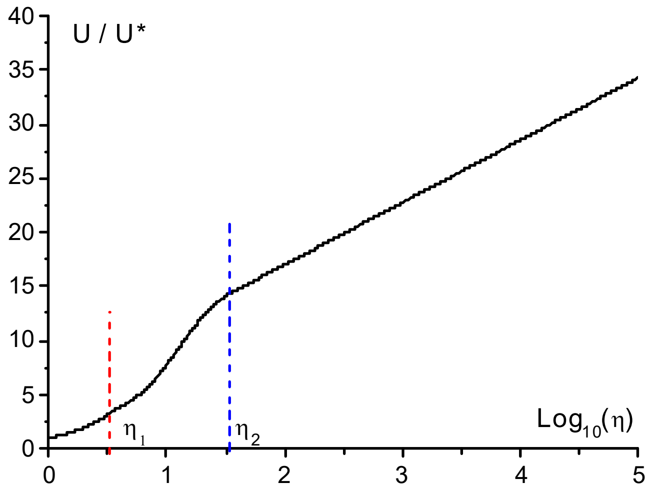

Fig. 2 . Theoretical evolution of U/U* in terms of $Log_{10}(\eta)$

As it was written before, the turbulence disturbs the profile of the space charge density in the diffuse layer. In this first model we suppose that the turbulence homogenized the space charge density in the center region of the pipe. To calculate the radius of this region we must compared the radial fluctuant velocity of the flow with the relaxation velocity of the diffuse layer. From analysis of the relaxation velocity V_R of the diffuse layer we have found [5] that it can be expressed as follows:

$$V_R = \frac{\delta_0 \sigma}{\varepsilon} = \frac{\delta_0 \sigma_1}{\varepsilon}\frac{\sigma}{\sigma_1} = V_{R0}\ \sigma_+ \tag{6}$$

δ_0 being the diffuse layer thickness, ε the permittivity and σ the conductivity inside the diffuse layer (which is function of the space charge density), σ_1 being the intrinsic conductivity of the liquid . This relaxation velocity must be compared to the radial fluctuant velocity V'. In order to increase the calculus we have found an analytical law which fit correctly with the experimental data of V'/U* in terms of η . This law is:

$$\frac{V'}{U^*} = 0.171\left\{Argsh\left[0.3911\eta - sh(1.46)\right] + 1.46\right\} \tag{7}$$

from V'/U*=0 to V'/U*=0.9, then V'/U* remains equal to 0.9 for greater η. We can see in Fig.4 the evolution of this law compared with the experimental data [13].

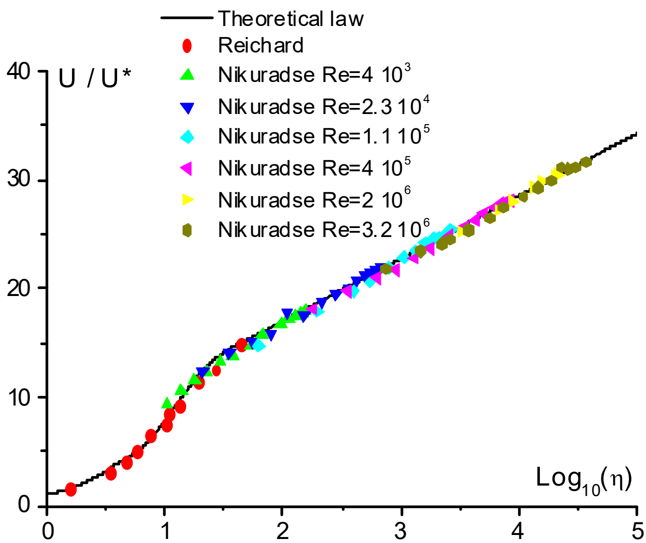

Fig.3 . Theoretical evolution of U/U* in terms of $Log_{10}(\eta)$ compared to experimental data.

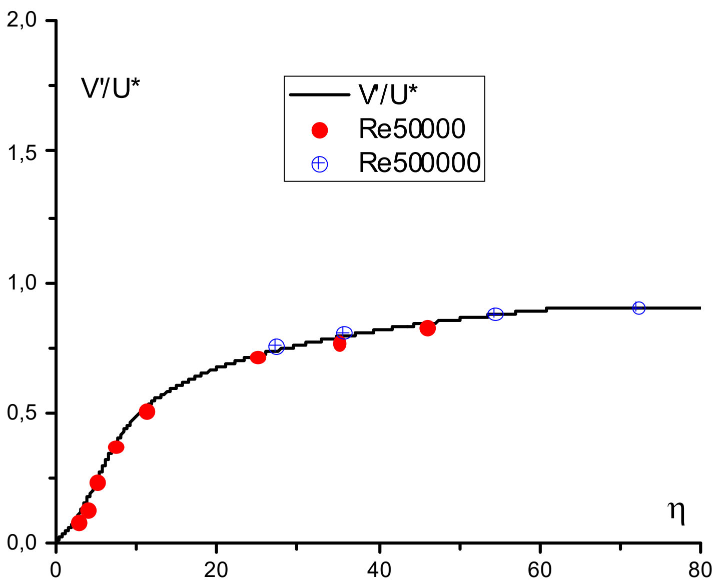

Fig.4. Analytical law of V'/U* compared to experimental data.

3) Diffuse layer perturbed by the turbulence

In Fig. 5 we show an example of a diffuse layer perturbed with a turbulent flow. In this example $R_+ = 1$, $\rho_{w+} = 11.3$, Re=4000 and $V_{R0}\,\mathrm{Re}^{7/8}/(U * R_+) = 0.18\ \ 10^3$. This last parameter determine the radial coordinate of the perturbed zone.

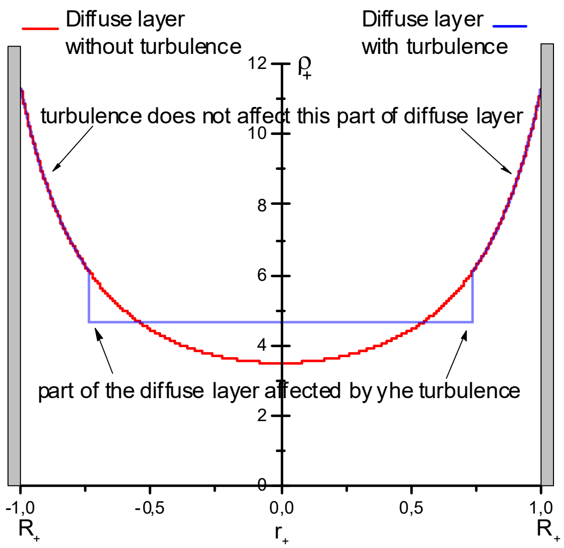

Fig. 5. Space charge density profile pertubed by turbulence.

Finally we plot in Fig.6 the ratio of the space charge transported by a turbulent flow Q_t over the space charge density transported by a laminar flow Q_l in terms of the Reynolds number Re and for different values of the space charge on the inner wall of the pipe ρ_{w+}.

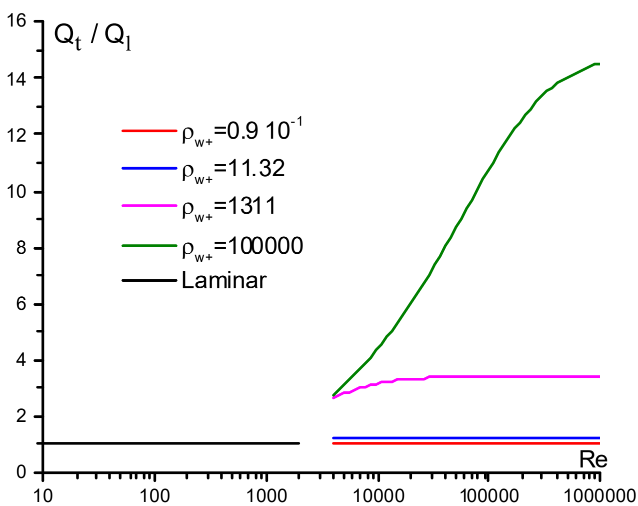

Fig. 6.Space charge density transported in terms of Reynolds number for several space charge on the wall.

A. Second model

1) Electrical quantities

In this model we used the equations of the diffuse layer existing in a pipe of circular cross section perturbed by a turbulent flow:

$$\begin{cases} (1 + DT_+)\,\overrightarrow{grad_+}\ \rho_+ + \sqrt{\rho_+^2 + 1}\,\overrightarrow{grad_+}\psi_+ \\ \Delta_+\psi_+ = -\rho_+ \end{cases} \tag{8}$$

With $DT_+ = DT / D0$ and DT is the turbulent diffusivity call also eddy diffusivity. This parameter varies with the radial coordinate and the Reynolds number. In the case of a pipe of circular cross section, the system of equations (8) becomes:

$$\begin{cases} (1 + DT_+)\dfrac{d\rho_+}{dr_+} + \sqrt{\rho_+^2 + 1}\,\dfrac{d\psi_+}{dr_+} = 0 \\ \dfrac{d^2\psi_+}{dr_+^2} + \dfrac{1}{r_+}\dfrac{d\psi_+}{dr_+} = -\rho_+ \end{cases} \tag{9}$$

This system of equation has no analytical solution whatever the value of ρ_+. All the calculus must be done numerically. The only assumption in this second model is to consider that the space charge density on the wall remains constant whatever the flow.

2) Flow Characteristics

The eddy diffusivity is given by the following equation:

$$DT = \frac{U^{*2}}{dU / dy}\left(1 - \frac{y}{R}\right) \tag{10}$$

As previously, to express the evolution of U/U* in terms of η we define three regions:

- The sub laminar region in which U/U*=η (this region corresponds to low Reynolds numbers Re) and the eddy diffusivity is consider equal to 0.
- The universal velocity distribution laws for very large Reynolds number cannot be applied because the eddy diffusivity DT computed from this law gives: DT=0.4yU*(1-y/R) which is equal to 0 on the axis of the pipe were the turbulence is very strong. So, we must find another expression. We have taken the expression given by Reichard [14]:

$$\frac{U}{U^*} = \frac{1}{k}\log\left\{\frac{3\eta\left[1 - \eta / \left(\mathrm{Re}\,U_+^*\right)\right]}{1 + C_2\left[1 - 2\eta\left(\mathrm{Re}\,U_+^*\right)\right]^2}\right\} + B \tag{11}$$

With $U_+^* = U^* / U_m$. In order to have a good agreement with the experiments and a good correspondence with the transition region, we choose k=0.395, C_2=2.06 and B must be computed for each Reynolds number but is always close to 4.

From the equation (11) we can, then, compute the value of the eddy diffusivity:

$$DT = \frac{k\nu}{4(1+C_2)} \operatorname{Re} U_+^* \left[1 - \left(1 - \frac{2\eta}{\operatorname{Re} U_+^*}\right)^2\right]\left[1 + C_2\left(1 - \frac{2\eta}{\operatorname{Re} U_+^*}\right)^2\right] \tag{12}$$

- Between these two regions exist the transition region in which we have introduced the same kind of law than in the previous model, just the coefficients are a little different:

$$\frac{U}{U^*} = 6\tanh(Log(\eta) - b) + 7.4 \tag{13}$$

b is computed for each Reynolds number in order to have a good agreement with the experiments and a good correspondence with the high Reynolds region. From equation (13) it is possible to compute the eddy diffusivity:

$$DT = \nu\left\{\frac{\eta}{6 - 6\tanh^2(Log(\eta) - b)} - 1\right\} \tag{14}$$

Remark: The maximum value for y is R, thus the maximum value for η is $\eta_{\max} = U_+^* \operatorname{Re}/2$. We can see in Fig.7 the evolution of the eddy diffusivity. In this model the limits η_1 and η_2 change with the Reynolds number and must be compute.

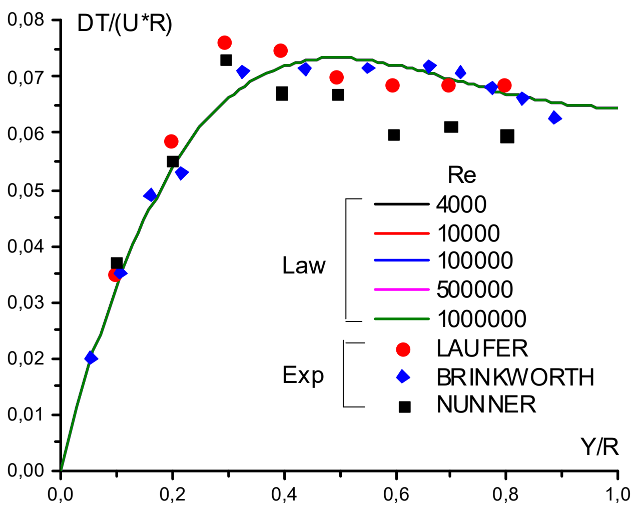

Fig.7. Evolution of the eddy diffusivity from the wall (y=0) to the center of the pipe y/R=1

We have plotted in Fig.8 U/U* in terms of η in the turbulent region for 5 different Reynolds numbers according to the law of Reichard limited to $\eta = \eta_{\max}$. In Fig. 9 these theoretical predictions are compared with the experiments made by Nikuradse and Reichard.

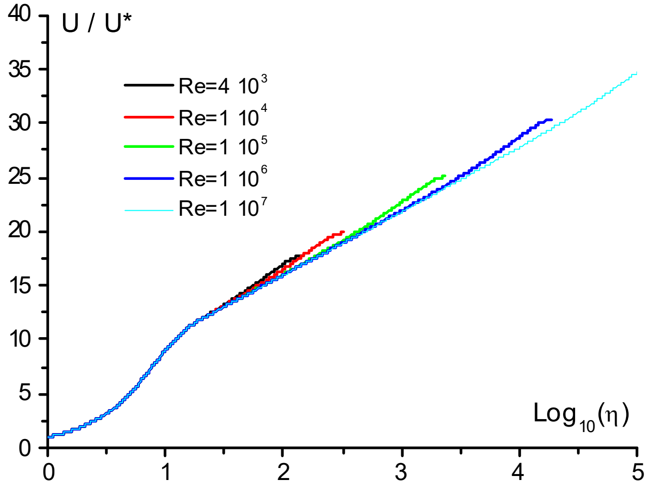

Fig.8. Theoretical law of the U/U* in terms of η.

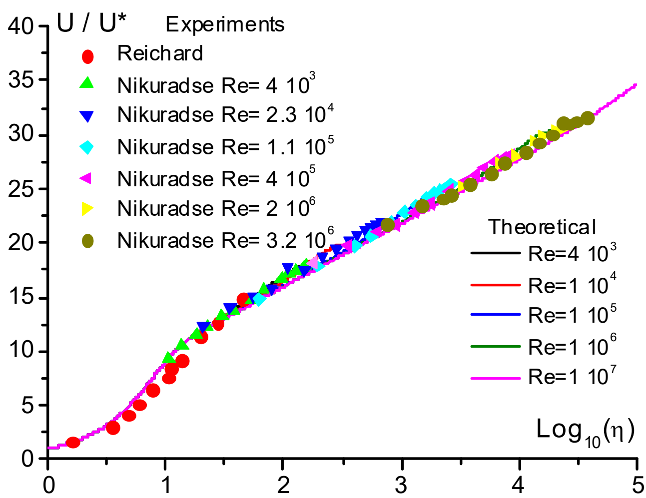

Fig. 9. Theoretical and experiments of U/U* in terms of η

We can see in Fig. 10 the evolution of the space charge density perturbed by the turbulence predicted by this model compared to a diffuse layer unperturbed. It is clear that this second model is more progressive but the total charge in the diffuse layer must evolve.
Finally we have plotted in Fig. 11 the space charge transported by the flow predicted by this model. It is much more important for turbulent flows than the predictions given by the first model, this is partly due to the fact that the total charge in the diffuse layer is greater than in the previous model.

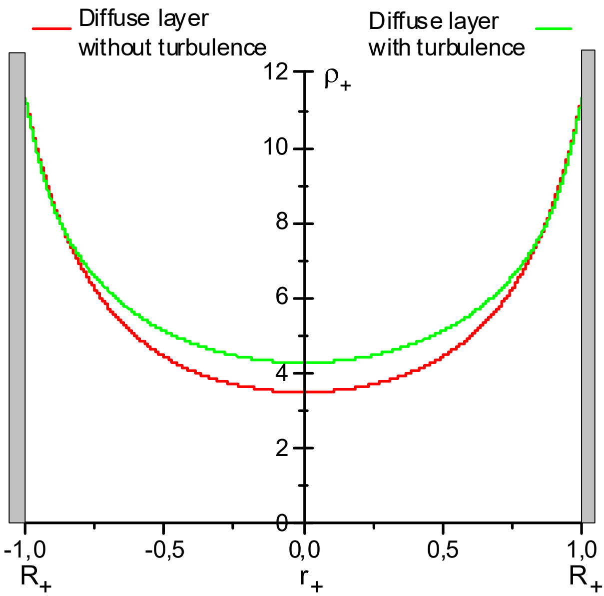

Fig.10. Evolution of the space charge density in the diffuse layer, with and without turbulence.

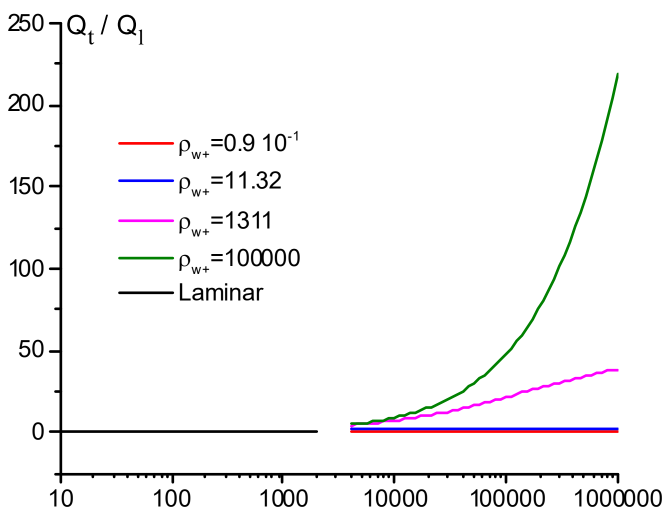

Fig. 11. Space charge density transported predicted by the second model.

III. Conclusion

In this paper we have presented two models of effects of turbulence on a diffuse layer. The first one using the hypothesis of a whole charge remaining constant and comparing the relaxation velocity of the diffuse layer with the fluctuant velocity of the flow shows two regions one in the center is supposed homogenized by the turbulence and leads to a discontinuity of the charge. This model is much less CPU time consuming that the second one but not really satisfactory. The second one, probably more satisfactory because of the absence of discontinuity in the space charge profile in the diffuse layer leads to an important wall current at the jump laminar turbulent.

Finally the two models give rather different results concerning the charge transported much more that it was expected in previous studies.

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