A Non-linear Model of Sensitivity Matrix for Electrical Capacitance Tomography

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Abstract—**The estimation of the permittivity distribution within a given volume by Electrical Capacitive Tomography relies on a relationship between the measured signal and the permittivity distribution, known as the sensitivity matrix. We show that relative permittivity variations must be lower than 25% to obtain errors smaller than 10% with the model whereas the permittivity can be scaled by up to 3.6 for the same error range with the proposed non- model. The errors remain below 15% over the whole range of permittivity.**

I. Introduction

Electrical Capacitance Tomography (ECT) is a tool for estimating the permittivity distribution of materials located in an area, generally a pipe [1]. An ECT sensor is a capacitive sensor comprising a set of electrodes surrounding the sensitive volume in which the permittivity distribution is estimated from the measurements of the capacitance between the electrodes. It is therefore necessary to accurately model the influence of the permittivity distribution inside the ECT sensor on the measured capacitances to reach reliable estimations. This influence is known as the sensitivity matrix which links the signal variation on one electrode to the permittivity variation in a given volume element inside the ECT sensor sensitive volume. The sensitivity matrix is however ly defined [2] leading to large errors when the permittivity variation exceeds few percents of its nominal value.

In this paper we propose a non- model for the sensitivity matrix based on physical considerations that approximate the limit of the capacitance variation when the permittivity in a given volume element inside the ECT sensor sensitive volume tends toward infinity. In the first section of the paper the physical background of ECT sensors is reminded and the usual linear-model of sensitivity matrix is described. In the second section, numerical experiments are presented showing the great precision of the linear model for small variations of the permittivity distribution inside the sensor sensitive volume, and the large error made for larger variations of the permittivity distribution. It is shown that simple assumptions makes it possible to evaluate the maximal signal variation and thus to estimate with a simple non-linear model the signal variation for all permittivity variations with limited errors over the whole range of permittivity.

II. Physical background of ECT

Electrical Capacitance Tomography is based on the influence of the material permittivity on the capacitance between the electrodes of the ECT sensor.

A. Generality

The charges on the electrodes of any capacitive sensor depend on the voltage applied to each electrode by

$$Q_i = c_{ij} V_j, \tag{1}$$

where Q_i is the charge quantity on electrode i, c_{ij} is the capacitive tensor and V_j is the voltage applied to electrode j. Notice that in tensorial notation, a sum is insinuated over all values of any subscript that appears twice in a term. In (1), there is a sum over all values of j, that is to say over all electrodes.

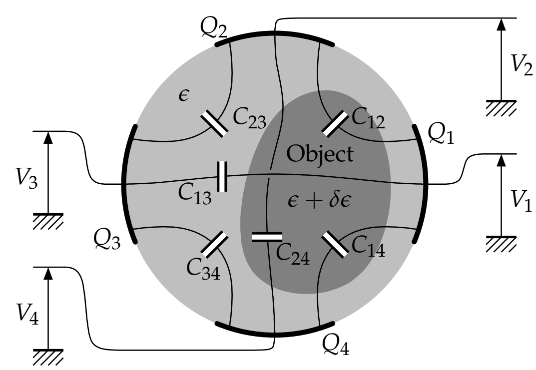

Figure 1. Simple Electrical Capacitance Tomography sensor showing all inter-capacities C_{ij} between the 4 electrodes of the sensor. The nominal permittivity is ϵ and the object permittivity is $\epsilon + \delta\epsilon$.

It can be shown that the capacitive tensor coefficients c_{ij} can easily be expressed form the inter-capacities C_{ij}

connecting electrode i to electrode j by

$$c_{ij} = \begin{bmatrix} \sum_j C_{1j} & -C_{12} & -C_{13} & \cdots & -C_{1N} \\ -C_{12} & \sum_j C_{2j} & -C_{23} & \cdots & -C_{2N} \\ -C_{13} & -C_{23} & \sum_j C_{3j} & \cdots & -C_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -C_{1N} & -C_{2N} & -C_{3N} & \ldots & \sum_j C_{Nj} \end{bmatrix} \quad (2)$$

Since there are at most $N(N-1)/2$ different inter-capacities for a N-electrode sensor, only $N(N-1)/2$ independent data can be obtained at most, for instance permittivity at $N(N-1)/2$ positions.

A variation in the environment of a capacitive sensor results in a signal which verifies for the electrode i

$$\delta Q_i = \delta(c_{ij}V_j) = \delta c_{ij}V_j + c_{ij}\delta V_{j\neq i} + c_{ii}\delta V_i. \quad (3)$$

In short-circuit measurement conditions, $\delta V_i = 0$ and the signal is the variation of charges δQ_i on electrode i. In open-circuit measurement conditions, $\delta Q_i = 0$ and the signal is the variation of voltage δV_i on electrode i. In both measurement conditions the signal results from the voltage variation δV_j on the other electrodes ($i \neq j$) and/or from a capacitive tensor variation δc_{ij} due, for instance, to a local variation of permittivity. Therefore both measurement conditions are not independent and it is possible to switch from one to the other using the Thevenin-Norton transforms:

$$\delta Q_i \equiv -c_{ii}\delta V_i. \quad (4)$$

B. Capacitive tensor coefficients

Capacitive tensor coefficients can be advantageously estimated by energy considerations. The energy W of an electrostatic system at equilibrium is the sum of the energy held by each electrode, that is to say half the charge quantity on the electrode multiplied by its voltage [3]:

$$W = \tfrac{1}{2}Q_iV_i = \tfrac{1}{2}c_{ij}V_iV_j. \quad (5)$$

Energy W can also be expressed as the integral over space of the energy density $\frac{1}{2}\epsilon E^2$, where ϵ is the permittivity and $\vec{E}$ the electric field. One has

$$W = \tfrac{1}{2}\int \epsilon\, E^2 \,\mathrm{d}v. \quad (6)$$

Considering a N-electrode ECT sensor, the electric field can be conveniently decomposed into the sum of the contribution of each electrode by introducing $\vec{\xi}_i$, the electric field produced by electrode i when polarized to 1 V while all other electrodes are grounded. One obtains

$$\vec{E} = V_i\vec{\xi}_i, \quad (7)$$

thus (6) becomes

$$W = \tfrac{1}{2}\int \epsilon\,(V_i\vec{\xi}_i)\cdot(V_j\vec{\xi}_j)\,\mathrm{d}v \quad (8)$$

and the identification between (5) and (8) leads finally to

$$c_{ij} = \int \epsilon\,\vec{\xi}_i\cdot\vec{\xi}_j \,\mathrm{d}v. \quad (9)$$

As far as capacitive sensor variation δc_{ij} is concerned, it has been demonstrated [4] that the variation of charges δQ_i in short-circuit conditions resulting from a small local permittivity variation $\delta\epsilon$ is at first order

$$\delta Q_i = \int \delta\epsilon\,\vec{E}\cdot\vec{\xi}_i \,\mathrm{d}v. \quad (10)$$

Using (7) for the electric field, one easily obtains δc_{ij} by identification:

$$\delta Q_i = V_j \int \delta\epsilon\,\vec{\xi}_i\cdot\vec{\xi}_j \,\mathrm{d}v = V_j\delta c_{ij}. \quad (11)$$

Field $\vec{\xi}_i$ can be seen as the influence of electrode i in the capacitive sensor. Therefore it can be called the sensitivity field of electrode i and plays a similar role than the radiation diagram of an antennae.

C. Sensitivity matrix

The reconstruction of the permittivity distribution in the ECT sensor volume depends on measurements and therefore relies on the influence of the permittivity in the signal. That influence is the signal sensitivity matrix S_{ij} and is generally defined as

$$S_{ij}V = \frac{\partial m_{ij}}{\partial \epsilon}, \quad (12)$$

where m_{ij} denotes the measurement at electrode i when electrode j is under the voltage V while other electrodes are grounded. Since ϵ depends on position, it is convenient to decompose the sensor sensitive volume into small elements δv_k. The permittivity is assumed uniform in each element k and can vary from ϵ to $\epsilon + \delta\epsilon^{\max}$. In that case the following linear expression is often used [2], [5]:

$$S_{ijk} = \frac{\delta v^{\max}}{\delta v_k} \times \frac{c_{ij}(\epsilon + \delta\epsilon_k^{\max}) - c_{ij}(\epsilon)}{\delta\epsilon^{\max}}. \quad (13)$$

where $\delta v^{\max}$ corresponds to the volume of the larger element. Expression (13) corresponds to the capacitance variation between electrodes i and j when the permittivity in element k is $\epsilon + \delta\epsilon^{\max}$ while the permittivity in others elements is ϵ, normalized by the permittivity variation $\delta\epsilon^{\max}$ and the volume δv_k of element k. From Equation (9) and (11), S_{ijk} can be expressed as

$$S_{ijk} = \frac{\delta v^{\max}}{\delta v_k} \times \int_{\delta v_k} \vec{\xi}_i\cdot\vec{\xi}_j \,\mathrm{d}v = \delta v^{\max} \times <\vec{\xi}_i\cdot\vec{\xi}_j>_k, \quad (14)$$

where $<\vec{\xi}_i\cdot\vec{\xi}_j>_k$ is the mean value of $\vec{\xi}_i\cdot\vec{\xi}_j$ over element k. The dot product $\vec{\xi}_i\cdot\vec{\xi}_j$ is therefore the sensor sensibility density.

D. Reconstruction algorithm

The reconstruction of the permittivity distribution in the sensor sensitive volume can be made by various algorithms [6], [5] which all rely on the sensitivity matrix S_{ijk}. The iterative Landweber algorithm is one of these algorithms that gives good results despite being relatively slow. That iterative algorithm is based on the optimization of the criterion J defined as

$$J = (m_{ij} - S_{ijk} V \hat{\epsilon}_k)^2, \tag{15}$$

where V is the voltage applied during the measurement. That criterion is minimized when the estimated permittivity distribution $\hat{\epsilon}_k$ produces the estimated signals $S_{ijk} V \hat{\epsilon}_k$ as close as possible to real measurements m_{ij}. The iterative algorithm consists then in iterative minimization of criterion J for instance by

$$\hat{\epsilon}_\ell^{n+1} = \hat{\epsilon}_\ell^n - \frac{\alpha}{2}\frac{\partial J}{\partial \hat{\epsilon}_\ell} = \hat{\epsilon}_\ell^n + \alpha\, S_{ij\ell} V\, (m_{ij} - S_{ijk} V\, \hat{\epsilon}_k^n). \tag{16}$$

Coefficient α is used to adjust speed of convergence. It is obvious from (15) and (16) that the better the sensitivity matrix S_{ijk}, the better the results of the reconstruction algorithm.

III. Discussion around the sensitivity matrix

A. Limitation of the linear model

For estimating the accuracy of the sensitivity matrix various situations have been simulated. One of them is presented in Figure 2. The ECT sensor is made of 8 evenly distributed electrodes around a cylinder and a grounded electrode enclosing all measurement electrodes. The measurement electrode filling factor is 50%, that is to say that the gap between electrodes is equal to the electrode width. The nominal permittivity is imposed at $\epsilon = 10\epsilon_0$, where ϵ_0 is the vacuum permittivity, to emphasize the influence of both much larger and much smaller object permittivity $\epsilon + \delta\epsilon$ on measurements. It is worth noting that taking any other value for the nominal permittivity ϵ would lead to the same conclusions. The object permittivity has been varied from $\epsilon/10$ to 10ϵ in all numerical experiments.

Figure 3 shows the variation of the first 8 measurements for one of the situations along with the linear model in dotted lines. It can be noticed that the linear model gives a very accurate estimation of the signal variation close to the nominal permittivity, that is to say for $\delta\epsilon/\epsilon$ close to zero. However as the permittivity variation $\delta\epsilon$ increases, a discrepancy appears since the signal variation δm rapidly reaches its maximal value while the signal variation estimated from the linear model continues to increase. Therefore the error is very large when the permittivity is too different from the nominal one. Though the errors differ from one situation to another, the overall relative error is less than 10% for all significant signal variations for $|\delta\epsilon/\epsilon| \lesssim 25\%$.

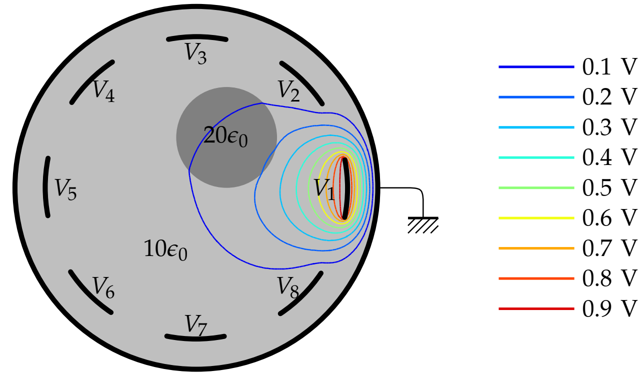

Figure 2. Example of one of the simulated situations with all electrodes grounded except $V_1 = 1\,\mathrm{V}$. The sensor radius is 10 cm and the object permittivity is twice the nominal one.

B. Proposed non-linear model

In order to take into account the non linearity of the measurements with permittivity, it is interesting to estimate the maximal value $m_{ij}^{\max}$ of all measurements m_{ij}. That maximal value is obtained when the permittivity tends toward infinity so that the object can be considered as a floating electrode. As a consequence both the electric field inside the object and the net charge quantity on the object surface are zero. Therefore the electrical energy that was at the position of the object has been redistributed between all measurement electrodes and, because that energy is limited, signals can only vary to a maximal value.

The redistributed energy is however not evenly shared out between all measurement electrodes. As a hypothesis it can be assumed that the redistributed energy is shared out between the electrodes with a coefficient corresponding to the influence of the electrodes at the object position. Equation (1) with (9) indicate that the fraction of charges $Q_i^{\mathcal{V}}$ on electrode i due to the part of space $\mathcal{V}$ corresponding to the object volume is

$$Q_i^{\mathcal{V}} = V_j \int_{\mathcal{V}} \epsilon\, \vec{\xi}_i \cdot \vec{\xi}_j \,\mathrm{d}v. \tag{17}$$

Removing that fraction of charges would be similar to a charge variation $\delta Q_i = -Q_i^{\mathcal{V}}$ on electrode i, and thus to a permittivity variation $\delta\epsilon = -2\epsilon$ after (11) since permittivity would have varied from ϵ to $-\epsilon$. Then assuming that the signal gain due to the energy redistribution caused by the infinite permittivity of the object is opposite to the signal loss when the part of space of the object is not taken into account, one gets the maximal variation of the signal $\delta Q_i^{\max}$ on electrode i as

$$\delta Q_i^{\max} \approx V_j \int_{\mathcal{V}} 2\epsilon\, \vec{\xi}_i \cdot \vec{\xi}_j \,\mathrm{d}v = 2\epsilon V_j S_{ij}(\mathcal{V}) \tag{18}$$

where $S_{ij}(\mathcal{V})$ is the sensitivity matrix related to the object volume. As a result the slope and the maximal value

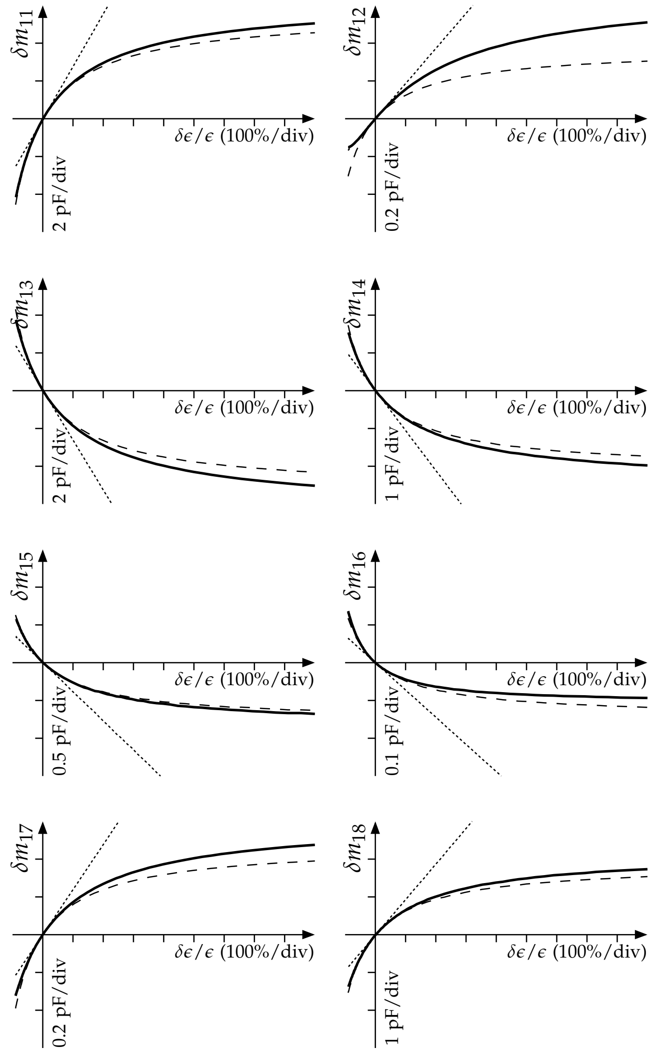

Figure 3. Variation of the first 8 measurements as a function of the relative permittivity variation $\delta\epsilon/\epsilon$ (solid line) along with the linear model (dotted line) and non-linear model (dashed line).

of δm_{ij} both directly depend on the sensitivity matrix $S_{ij}(\mathcal{V})$.

As the variation δm_{ij} are relatively smooth, a simple fitting low can be used to estimate the measurements, for instance

$$\delta m_{ij} \approx \frac{\delta\epsilon\, \delta m_{ij}^{\max}}{\delta\epsilon + 2\epsilon} \tag{19}$$

which value is $\delta m_{ij}^{\max} = 2\epsilon\, S_{ij}(\mathcal{V})\, V$ when $\delta\epsilon$ tends to infinity and slope is $S_{ij}(\mathcal{V})\, V$ at $\delta\epsilon = 0$.

Figure 3 shows the first 8 measurement variations along with the non-linear model in dashed lines. It can be noticed that the non-linear model follows the tendency of the measurements hence reducing the error over a large range of permittivity variation. The relative error is less than 10% for all significant signal variations for an object permittivity up to 3.6 times the nominal one and remain constrained within ±15% for any permittivity variations.

IV. Conclusion

A non-linear model for the sensitivity matrix of Electrical Capacitive Tomography sensors has been proposed in order to reduce the error of the estimated signal in reconstruction algorithms. The non-linear model is based on the approximation of the maximal signal the sensor can measure if the object to detect were replaced by a floating electrode. A simple fitting law taking into account the slope of the measurements around the nominal permittivity and the maximal signal variation is then applied to express the sensitivity matrix for all permittivity values. It is shown that the estimation errors of the non-linear model are very small compared to the estimation errors made by the linear model, making it possible to improve reconstruction of the permittivity distribution with ECT sensors.

References

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